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TIDAL PHENOMENA AT INLAND BOREHOLES NEAR CRADOCK.

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(Read June 19, 1912.)

(Plates III.–VI.)

In the following pages the terms "artesian well," "subartesian well," and "potential level" are used in the following sense :—

An artesian well is a bored well in which the water rises naturally to the surface of the ground and flows.

A subartesian well is a bored well in which the water rises above the level at which it was first encountered to a level within pumping reach of the surface.

The potential level of a well is the level at which the water stands when piped up as high as it will reach without recourse to pumps or siphons.

An artesian well might thus be described as a well whose potential level is above the level of the ground surface, and it will be observed that I do not use the term "artesian" as implying any particular theory of the origin of the water pressure.

During the last twenty years a large amount of boring for water has been carried on in the interior of Cape Colony. Very little was done in this direction previous to the year 1893. During the sixteen years, 1893 to 1909, the Cape Government subsidised water-boring enterprise to an extent probably unparalleled elsewhere in the world.

A very large Government water-boring establishment equipped with fifty to sixty drills of various types was gradually built up, and about half the cost of the work done was met by grants from the public exchequer. In connection with this expenditure of public money detailed records of most of the boreholes were carefully filed in the Government offices and generalised information based on these records was published in various Annual Reports of the Government Departments concerned in the work. During the later years of the period mentioned the policy of the Government was directed towards the reduction of the Government boring establishment and the encouragement of private boring contractors—the general object being the gradual cessation and final abolition of Government aid.

In 1910 the Government boring establishment was finally abolished, and the flourishing private boring industry that had sprung up was left to carry on the work on commercial lines. Information regarding the progress of boring since that date has necessarily become scanty and largely unreliable, as the detailed records are no longer filed in public offices.

The general results of the work of the Government drills from 1893 up to the beginning of 1910 are summarised in the Annual Report of the Public Works Department for 1909–10 as follows:—

Total number of holes bored by Government drills	5,596 holes
Total number of feet bored	405,355 feet
Number of holes in which water was found	4,326 holes
Number of holes yielding over 1,000 galls. per diem	3,856 holes
Estimated total yield of artesian or "flowing" water	18,663,944 galls. per day
Estimated total yield of subartesian water rising to within pumping reach	35,498,775 galls. per day
Estimated total yield of all holes	54,162,719 galls. per day

It will be seen from the first two lines that the average depth of these holes is only 72 feet, and as a matter of fact very few holes of over 500 feet have been made in Cape Colony. The few deeper holes have in most cases been unsuccessful. In the selection of sites for the boreholes various considerations have prevailed. The convenience of the farmer has often been the dominant consideration. In many cases a "dowser" has been employed by the farmer to select sites. In many other cases (probably about 50 per cent. of the whole) the farmer has left the selection of the sites to the Government official in charge of the drill. These drill foremen all obtained a little elementary instruction in geology mainly directed to enable them to recognise and trace the dolerite dykes which play an important rôle in the Karroo System, and they were instructed to avoid boring in dolerite but to select sites if possible within a few hundred yards of a dolerite dyke. This principle probably guided the majority of their selections.

The statistics published by the Chief Inspector of Water-Boring show that in general about 75 per cent. of the holes bored in the Karroo System were successful in yielding over 1,000 galls. of water a day. When rocks of pre-Karroo age were bored the percentage of success was much smaller.

The Reports further show that of the wells bored in the Karroo System about 25 per cent. yielded "flowing" or artesian water, while about 50 per cent. yielded subartesian water rising to within pumpable reach of the surface.

In nearly all cases in which water was struck the water rose in the borehole some distance above the stratum in which the water was first encountered.

During the last eight years a large number of holes have been bored in addition to these Government drillings. The Report of 1909 already quoted refers to the existence at that date of 102 private boring contractors in the Colony, and the Chief Inspector of Water-Boring in his Report for 1907 states that these boring contractors were putting down boreholes at the rate of 1,400 a year. As a rule these contractors bore only in the Karroo System or in localities where past experience has shown a considerable prospect of success. I estimate that altogether at the present day there must be over 10,000 successful boreholes in Cape Colony.

I have at various times visited a large number of these boreholes both during their construction and afterwards. It has become obvious to me that the vast majority of them merely tap shallow supplies dependent on the local rainfall, and that they are nearly all liable to give a diminished output or in some cases to dry up altogether after local droughts.

The temperature of most of these waters is approximately the same as the probable rock temperature within a few hundred feet of the surface. For purposes of irrigation the quantity of water yielded is, with few exceptions, insignificant, but for the purpose of watering stock and for domestic use these wells are of the utmost value to the country.

Practically the only attempt at a general theory of the artesian pressure, movement, and position of underground water in the Karroo System that has been hitherto recognised is what might be termed the "dolerite intrusion" theory.

This theory is fully explained in its various applications by Mr. H. P. Saunders in a Government Blue Book published in 1897 and entitled "Underground Water-supply of the Colony and the Cape of Good Hope." The substance of this theory is that the water enters the more pervious layers of Karroo sediments at their outcrops on elevated ground and percolates along these layers through pores and cracks to lower levels until stopped and dammed back by one of the numerous impervious dolerite intrusions. That underground dams of waterlogged rock are thus produced in which the water, under hydrostatic pressure, is held down by superincumbent shale or other impervious strata and held back horizontally by impervious dolerite dykes.

I have in the course of my wanderings seen much to support the opinion that this theory embodies a considerable amount of truth, but I have also come to the conclusion that it is far from containing the whole truth. Its application is confined to the obviously shallow local supplies of water, and it fails to meet the case of the few hot springs and hot-water wells that also occur in the Karroo System.

My observations have led me to hold the view that the waters of the Karroo System can be sharply divided into two great classes. On the one hand, those waters that are superficial in position, having variable temperatures of from 66 to 70° F. as in the vast majority of the boreholes and natural springs of the country; and on the other hand, those waters that are characterised by high temperatures of from about 75 to nearly 100° F., by the presence of sulphuretted hydrogen and methane gas, and generally also by the presence of a curious bacterium which flourishes in sulphurous waters. This deep-seated water is best known at a number of points where it emerges through natural springs, as at Aliwal North and Cradock.

As nearly all the boreholes extend to shallow depths, it has happened (as was to be expected) that very few have succeeded in tapping this deep-seated sulphurous water.

The happy accident of a shallow borehole having intersected a fissure leading down to great depths or a porous stratum fed by such fissures accounts for the few exceptions. Irregular fissures leading down to great depths also supply the only feasible explanation of the occurrence of such natural springs of thermal water as are seen at Aliwal North and Cradock.

When this contrast of the two classes of Karroo water had struck me, I began to feel that a careful study of the sulphurous occurrences would be most likely to lead to a better theory of the underground water of the Karroo, and my attention has since then been largely concentrated on that class, though intermittently, as the time at my disposal for such work was confined to infrequent long academic vacations, and each visit to the interior involved my spending three or four days in railway trains.

The position of the group of boreholes which I have studied in most detail is on a farm formerly called "Driefontein," but now known as "Tarka Bridge." It lies about 15 miles to the south-east of the town of Cradock, and on it the junction of the Tarka River and Fish River occurs. The arable land forming the greater portion of the farm lies between 2,700 and 2,800 feet above sea-level, while the mountainous portions on the south-east and east rise much higher—probably exceeding 4,000 feet in parts.

The geological structure of the farm is very simple. A series of shales and sandstones with a very slight northward dip and a few intrusive dykes and sills of dolerite are the principal features. The only fossil

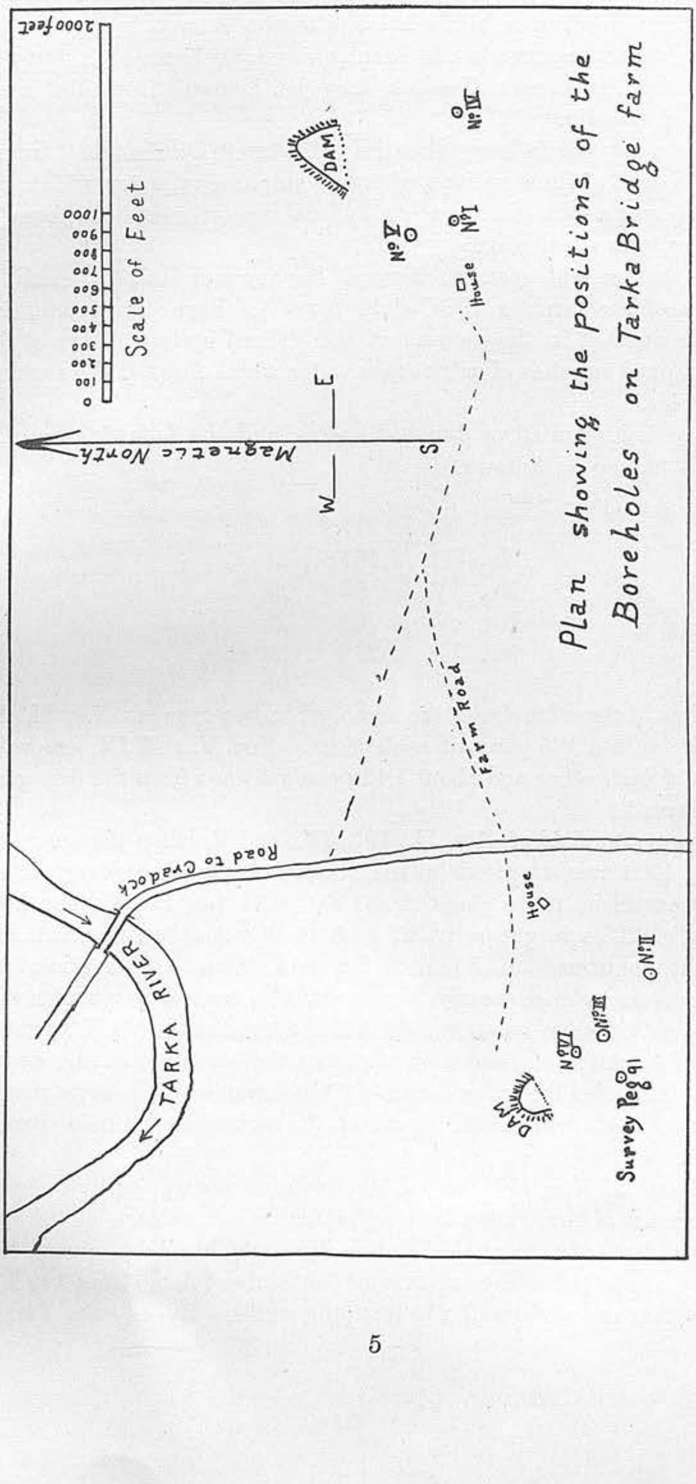

Plan showing the positions of the Boreholes on Tarka Bridge farm
Scale of Feet
0 100 200 300 400 500 600 700 800 900 1000
2000 feet
Magnetic North
W E S
DAM
No. IV
No. V
No. I
House
Farm Road
Road to Cradock
TARKA RIVER
House
No. II
No. III
No. VI
DAM
Survey Peg 91

FIG. 1.

which I found was a glossopteris impression on a fine-grained sandstone. The sedimentary series certainly belongs to the Karroo System, and the probability that it represents a horizon considerably lower than that of the reptile-bearing beds near Cradock may be inferred from the general direction of the dip.

The farm appears to have taken its old name "Driefontein" from the occurrence on it of three springs of warm sulphurous water which seem to have enjoyed a somewhat restrictedly local reputation of being beneficial to sufferers from rheumatism.

Since the present owners, Messrs. Rayner and Roberts, came into possession of the farm in 1903, eight boreholes have been made, six of which are situated in the vicinity of the natural springs. Five of these six have tapped supplies of sulphurous water which flows at the surface of the boreholes.

The boreholes are all of 6-inch diameter, and the following boreholes have been named as follows:—

No. II.	is 204	feet deep
No. III.	is 167	"
No. IV.	is 65	"
No. V.	is 65	"
No. VI.	is 225	"

The sites of these boreholes are arranged in two groups: Nos. II., III., and VI. are within 185 yards of each other. Nos. V. and IV. are within 225 yards of each other and about 1,420 yards distant from the first group. (See Diagram 1.)

The aggregate yield of Nos. II., III., IV., and V. when they were first opened in 1903 was reported by the officers of the Government Public Works Department to be about 1,250,000 galls. per twenty-four hours. The initial yield, as is generally the case in this country, was not maintained, but continued to diminish for some time. This diminishing process appears to have nearly, if not entirely, ceased now, and a fairly constant yield to have been established. In the case of No. V., however, a periodic fluctuation of yield soon attracted the attention of the owners. In all five boreholes the water comes up accompanied by a large quantity of inflammable gas which bubbles out at the surface and smells strongly of sulphuretted hydrogen.

In the case of Nos. III. and VI. the escaping gas can easily be ignited by the mere act of throwing a burning match on the surface of the water as it escapes from the borehole. If the borehole has been closed for a quarter of an hour before the experiment the lambent flame from 1 to 2 feet high plays over the surface of the escaping water. By day the flame is bluish like that of a Bunsen burner, but by night the flame appears rather yellow (sodium flame), presenting a very weird appearance. The flame, after burning thus for a minute or two, is easily extinguished by the slightest puff of wind.

My personal observations of these boreholes began in Jauuary, 1905. About the end of 1904 I heard that Messrs. Rayner and Roberts had noticed a serious periodic variation in the apparent yield of borehole No. V. They said the variation was like a "tide," and although I was very sceptical about the propriety of this term as applied to the fluctuation, I thought this report was worthy of some inquiry.

In January, 1905, I spent a fortnight on the farm and made the following series of measurements.

1. *Temperature of the Water issuing from the Boreholes.*—The temperatures were observed on a maximum mercurial thermometer graduated in Fahrenheit degrees. Tenths of a degree were estimated by eye.

The National Physical Laboratory Certificate dated April, 1903, gives the correction at 72° and 82° as $-0{\cdot}1°$.

In each case the mercury column was fully immersed in the water at the mouth of the borehole.

	Readings.	Corrected Readings.
Borehole No. II.	78·5° F.	78·4° F.
" III.	81·0° F.	80·9° F.
" IV.	77·6° F.	77·5° F.
" V.	80·5° F.	80·4° F.

During the fortnight I frequently took readings of the temperatures at various hours of the day and night, but was unable to find any variations of temperature during that period. The various readings at any one borehole always agreed within $\frac{1}{10}$ of a degree, and a few discrepancies were clearly within the limits of observation error.

II. *Yield of the Boreholes.*—The yields of the four flowing wells were measured from time to time by allowing the water to flow into an iron vessel of $11\frac{1}{3}$ galls. capacity and determining by means of a stop watch the time occupied in filling the vessel.

The following were the readings obtained:—

Borehole No. III. on Jan. 27	at 6.50 p.m.	gave $11\frac{1}{3}$ galls.	in 13	sec.
28	10.0 a.m.	"	13	"
"	11.35 a.m.	"	13	"
"	2·55 p.m.	"	13	"
"	7·0 p.m.	"	13	"

Borehole No. II. on Jan. 27	at 6.25 p.m.	gave $11\frac{1}{3}$ galls.	in 44	sec.
28	11.55 a.m.	,,	45	,,
,,	3.0 p.m	,,	44	,,
,,	6.55 p.m.	,,	45	,,

Borehole No. IV. on Jan. 28	at 9 a.m.	gave $11\frac{1}{3}$ galls.	in 45	sec.
,,	10.30 a.m.	,,	44	,,
,,	12.40 a.m.	,,	44	,,
,,	2.25 p.m.	,,	44	,,
,,	3.35 p.m.	,,	45	,,
,,	5.5 p.m.	,,	45	,,
,,	6.35 p.m.	,,	45	,,

Borehole No V.—

On Jan. 27	at 12 noon	gave $11\frac{1}{3}$ galls.	in 12	sec.	= 56·6	galls. per min.
,,	1 p.m.	,,	13	,,	= 52·3	,,
,,	5 p.m.	,,	15	,,	= 45·3	,,
28	8 a.m.	,,	17	,,	= 40·0	,,
,,	9 a.m.	,,	16	,,	= 42·5	,,
,,	10.30 a.m.	,,	14	,,	= 48·6	,,
,,	12.30 p.m.	,,	13	,,	= 52·3	,,
,,	2.20 p.m.	,,	13	,,	= 52·3	,,
,,	3.30 p.m.	,,	14	,,	= 48·6	,,
,,	5 p.m.	,,	$14\frac{1}{2}$	,,	= 46·9	,,
,,	6 p.m.	,,	15	,,	= 45·3	,,
,,	7.45 p.m.	,,	14	,,	= 48·6	,,
29	9.55 a.m.	,,	$16\frac{1}{2}$	,,	= 41·2	,,
,,	10.50 a.m.	,,	15	,,	= 45·3	,,

Thus it would appear that No. V. alone exhibits a notable variation in yield.

It will be observed that on January 28, between 8 a.m. and 8 p.m., the yield rises to a maximum in the neighbourhood of 1 p.m. and falls to a minimum about 6 p.m.

This No. V. borehole opened into the bottom of an iron tank about 2 feet square and 2 feet deep. An exit pipe of $5\frac{3}{4}$-inches internal diameter at the side of the tank conducted the water from the small tank A for a distance of about 15 feet into a larger and much deeper tank B. (See Diagram 2.) The water on emerging from this pipe had a drop of several feet through air before reaching the level of the water standing in B. A second exit pipe from the bottom of B led the water into a dam about 100 yards distant.

It was observed that the level of the water surface in the small tank A was always higher than the upper edge of the first exit pipe. This is easily intelligible when it is remembered that the water entered tank A through a vertical 6-inch diameter pipe and left by a $5\frac{3}{4}$-inch pipe, nearly horizontal, with a fall of only 3 inches in 15 feet.

As the level of the water surface was found to vary gradually, it occurred to me that a series of measurements of the height of this water surface would give an indication of the varying pressure at which the

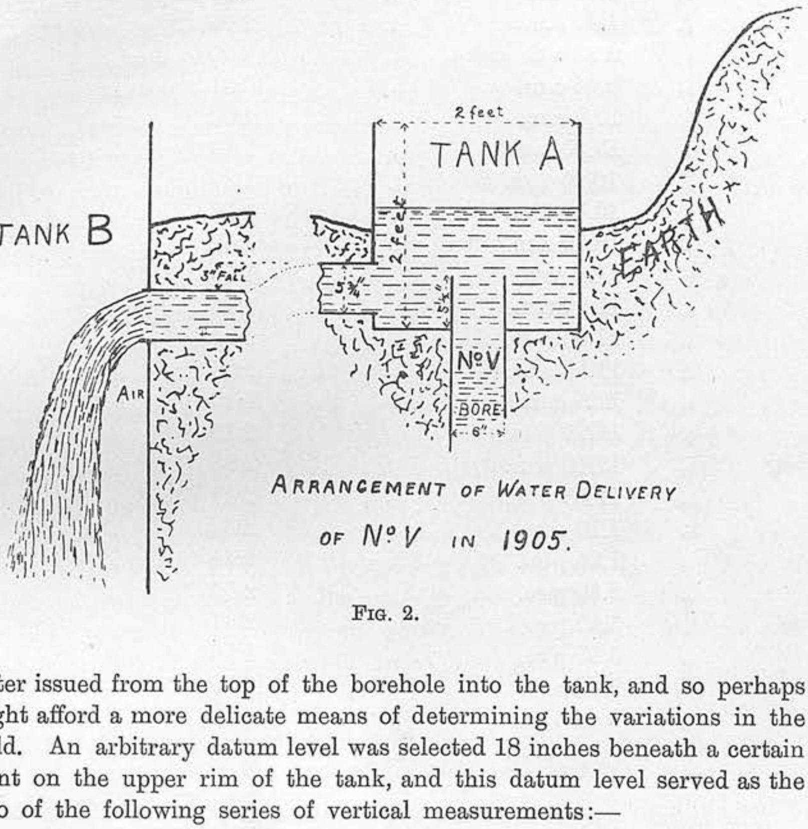


Fig. 2.

water issued from the top of the borehole into the tank, and so perhaps might afford a more delicate means of determining the variations in the yield. An arbitrary datum level was selected 18 inches beneath a certain point on the upper rim of the tank, and this datum level served as the zero of the following series of vertical measurements:—

On Jan. 28	at 8 a.m.	the level of the water	in tank A was	0·25 in.	above datum.
,,	9 a.m.	,,	,,	0·81	,,
,,	10.45 a.m.	,,	,,	1·62	,,
,,	12.30 p.m.	,,	,,	2·12	,,
,,	2.15 p.m.	,,	,,	2·00	,,
,,	3.30 p.m.	,,	,,	1·75	,,
,,	5.0 p.m.	,,	,,	1·50	,,
,,	6.0 p.m.	,,	,,	1·25	,,
,,	6.30 p.m.	,,	,,	1·31	,,

Jan. 28	at 7.20 p.m.,	height of water-level	1·37	in. above datum	
,,	7.45 p.m.	,,	1·47	,,	
,,	9.10 p.m.	,,	1·67	,,	
,,	10.10 p.m.	,,	1·75	,,	
,,	11.5 p.m.	,,	1·97	,,	
29	12.10 a.m.	,,	2·12	,,	
,,	1.15 a.m	,,	2·12	,,	
,,	1.45 a.m.	,,	2·25	,,	
,,	2.25 a.m.	,,	2·00	,,	
,,	9.45 a.m.	,,	0·81	,,	
,,	10.0 a.m.	,,	1·00	,,	
,,	10.10 a.m.	,,	1·19	,,	
,,	10.25 a.m.	,,	1·37	,,	
,,	10.40 a.m.	,,	1·50	,,	
,,	10.50 a.m.	,,	1·69	,,	
,,	11.4 a.m.	,,	1·56	,,	
,,	11.20 a.m.	,,	1·75	,,	
,,	11.33 a.m.	,,	2·00	,,	
,,	11.53 a.m.	,,	2·06	,,	
,,	12.6 p.m.	,,	2·12	,,	
,,	12.25 p.m.	,,	2·19	,,	
,,	12.40 p.m.	,,	2·25	,,	
,,	12.55 p.m.	,,	2·44	,,	
,,	1.10 p.m.	,,	2·37	,,	
,,	1.25 p.m.	,,	2·44	,,	
,,	2.30 p.m.	,,	2·50	,,	
,,	3.40 p.m.	,,	2·37	,,	
,,	5.55 p.m.	,,	1·56	,,	
30	9.30 a.m.	,,	0·19	,,	
,,	9.45 a.m.	,,	0·22	,,	
,,	10.30 a.m.	,,	0·75	,,	

The highest part of the inner edge of the exit pipe was 0·31 in. below the datum line.

The curve obtained by plotting these measurements on squared paper like the curve of the yield shows a maximum on January 28 in the neighbourhood of 1 p.m. and a minimum about 6 p.m.

Further, it may be said to show in a general way in the course of a day and a half three maxima and three minima at roughly equal intervals of six or seven hours.

It was now apparent that the best means of obtaining further information about the periodic variation of yield would be some form of clock-driven, self-recording instrument registering the fluctuations of water-level in tank A. Accordingly further attempts to measure these variations were deferred until such time as the recording apparatus could be constructed.

Meanwhile I directed my attention to the question why No. V. should be exhibiting marked variation of yield while Nos. II., III., and IV. were nearly constant, showing little if any variation, although all four were giving forth high-temperature water with strong sulphurous odour and inflammable gas.

I closed both No. V. and No. IV. No. V. was closed by means of a wooden plug, through which a round hole was bored and a few feet of glass tube inserted to fit tight and stand in a vertical position. The water at once mounted in the glass tube to about 5 inches above the datum line, and then in the course of an hour slowly rose 7 inches higher.

Thus the *potential level* of No. V. is less than a foot above the level at which the water usually stands in tank A.

About an hour after the two boreholes had been closed No. IV. was opened again. Meanwhile I closely observed the water-level in the glass tube at No. V. The instant of opening No. IV. was signalled to me by Mr. Rayner by means of a gunshot. Exactly 60 seconds after I heard the shot I noticed the water-level in the tube shiver and drop about $\frac{1}{12}$ inch. Then it steadily dropped about $\frac{3}{8}$ inch per minute for a few minutes, after which the rate of fall diminished slightly. Twenty minutes after the first shiver the level had fallen about $4\frac{1}{4}$ inches in the tube.

It was clear then that No. IV. and No. V. had a close connection.

I failed, however, to obtain any evidence of a similar relationship between No. V. and the distant boreholes Nos. III. and II. I afterwards found that No. III. reacted to the opening or closing of No. II.

With a levelling instrument and surveyor's staff I next determined the relative heights of the boreholes to be as follows :—

Datum mark in water	at No. V. is	3·53 feet above level of No. IV. orifice	
,,	,,	6·70 feet above orifice of No. II.	,,
,,	,,	16·72 feet above orifice of No. III.	,,

More recently I have been able to relate these levels with sea-level as follows. The Government Irrigation Department has made a survey traverse in connection with a proposed irrigation furrow which was to be made through the farm. This traverse connects with the railway survey levels at Mortimer, which latter survey of course goes down to the coast at Port Elizabeth.

Peg 91 of the Irrigation Survey is situated within 100 yards of No. III., and on levelling from this peg I found that No. III. lies 8·95 feet lower.

On inquiry I find that Peg. 91 is 2,706 feet above railway level sea datum.

The borehole levels then become—

No. II.	2,707	feet above sea-level
No. III.	2,697	„
No. IV.	2,710	„
No. V.	2,713·7	„

It has been already mentioned that the water of No. V. when piped upwards rose about a foot above the datum mark. This indicates an upward pressure of the water at the top of the borehole of less than ½ lb. on the square inch.

In the case of No. III. the pressure was much greater, and by closing that hole and balancing its pressure in a U-tube against a column of mercury I found that at first the pressure was equivalent to 10 inches of mercury column, rising during the first 5 minutes to 12·1 inches, and after 10 hours to 14·1 inches of mercury—a pressure of about 7 lbs. on the square inch. In other words, while the potential level of No. V. water was less than a foot above the borehole orifice, the potential level of No. III. was about 16 feet above the orifice. Incidentally we may also remark that the potential levels of No. V. and No. III. as thus determined were nearly the same, namely, 2,714½ and 2,713 feet above sea-level.

The foregoing data of relative levels and pressures seem to suggest a sufficiently plausible explanation for the comparative absence of very notable fluctuations in the yields of Nos. II. and III. The water of No. V. issues at so small a pressure that an increment of pressure amounting to an ounce or less on the square inch would naturally produce a noticeable effect, while a similarly small increment added to an already existing pressure of 6 or 7 lbs. on the square inch at No. III. might affect the yield there by an amount which might easily escape observation.

We also have the suggestion that if No. III. were piped up to the neighbourhood of its potential level it might be possible to detect fluctuations similar to those of No. V. Later it will be seen that this suggestion was carried out with results that confirmed the general truth of the above tentative reasoning.

On my return to Cape Town after my first visit to Tarka Bridge I had a self-recording instrument made there with such materials as I could find. Mr. Brackenbury Baily, of the Telegraph Department, kindly lent me a 4½-inch diameter revolving drum, with a fairly good spring-driven clock driving it through one revolution a day. Mr. W. H. Cottell, a highly skilled fine mechanic with the training of a chronometer maker, was at that time in the employment of the Public Works Department, and he carried out the construction of the machine under my direction.

The apparatus consisted of a large metal float firmly fixed to a vertical brass rod, the upper end of which was connected by a hinge joint to a horizontal brass arm lever of the first order about 19 inches long. At the end of the lever remote from the float was fixed a pencil, recording its vertical movement on the drum, which revolved on a vertical axis once a day.

The fulcrum support was placed near the middle of the lever. The pencil-point was found to be, as nearly as one could measure, 9 inches from the point of support, so that the pencil moved in an arc of a circle of

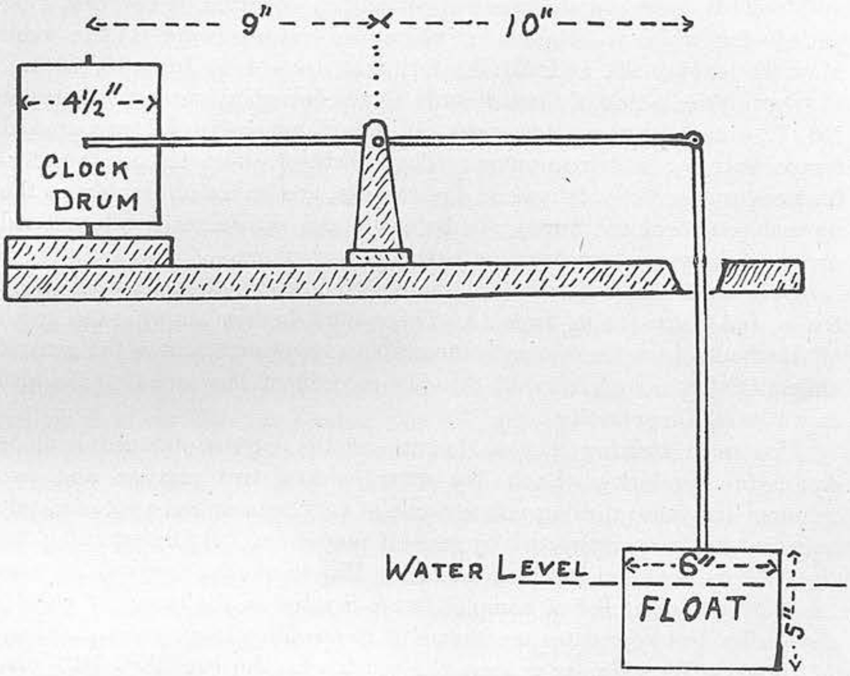


FIG. 3.

9 inches radius. The top of the float rod was found to be 10 inches from the point of support of the lever. Thus the actual vertical movement of the float was reduced on the record by about $\frac{1}{10}$ of its amount. As thus arranged an upward movement of the float was recorded by a downward movement of the pencil. The essential dimensions and general arrangement of the parts of the apparatus are exhibited in Diagram 3. The whole apparatus, with the exception of the float and part of the upright rod, was enclosed in a stout wooden box fitted with a plate-glass lid. A hole in the bottom of the box vertically under the hinge joint was fitted

with a thin brass collar, which served to keep the float rod vertical while effecting little frictional retardation of its vertical movements. When fixing the machine in position over the borehole care was taken to level the drum with a spirit-level, so as to ensure that its axis of rotation should be truly vertical.

As Messrs. Rayner and Roberts use the water of the boreholes for irrigation, and frequently have to regulate the flow to suit their requirements, it is only at certain limited periods that opportunities of using the recording apparatus occur.

The apparatus was fixed up over the small tank A at borehole No. V., and records were taken when opportunities occurred. The first long periods for which I obtained uninterrupted records were (1) the week May 22 to May 29, and (2) the fortnight June 4 to June 18, 1905.

During the taking of these records all the boreholes were closed except No. V., except for three occasions, on each of which No. IV. was opened temporarily for a few minutes. The effects of these temporary interferences are perfectly defined on the records, and make no trouble, as the normal course of the curve can be easily and satisfactorily interpolated owing to the very short duration of the perturbations.

See Plate III

Diagram 4 shows a tracing of the record sheet for the 24 hours' period 8 a.m. June 12 to 8 a.m. June 13. This record, besides showing the notch effect of one of the three interferences above mentioned, shows the general characteristics common to all the day records of the series. (See also curve A in Diagram 9.)

and Plate VI

The most striking general feature of the curves obtained is their wonderful regularity. Each day record shows two maxima and two minima, and these turning-points occur at very regular and approximately equal intervals, as estimated by general inspection. By determining the times of the first and last maxima in the May week and dividing the time interval by the number of complete wave-lengths on the record, I quickly obtained a first approximate estimate of the average wave period as about $12\frac{1}{2}$ hours. By a similar process the records for the June fortnight gave the same result.

On one side of the curve will be observed a large number of nearly vertical "hair" lines. Each of these "hair" lines records the escape of an unusually large gas-bubble, which on its arrival at the top of the borehole has impinged on the under surface of the float and caused a momentary upward jolt.

It is worthy of notice that in general the occurrence of these large gas-bubbles is much more frequent near the periods of high water than about low-water periods.

Another general feature of the records is the occurrence of a frill of minor fluctuations, with period varying from 5 to 20 minutes, and of an amplitude generally less than 2 millimetres. Occasionally also there appears here and there a wave-like movement of 3 or 4 millimetre amplitude and 1 or 2 hour period.

I have examined a number of records taken by the tide gauges at various points along the South African coast, and find that a frill of minor fluctuations of similar periods forms a characteristic feature of the marine tide curves, and I am given to understand that they are generally ascribed to marine seiche movements.

The amplitudes of the great $12\frac{1}{2}$-hour period waves were obviously subject to considerable variation. A general inspection brought to light the fact that these amplitudes attained a maximum of about $1\frac{1}{2}$ inches on June 17, which happened to be the date of full moon, and attained a minimum of less than $\frac{3}{4}$ inch about June 10, when the moon reached its first quarter. Another minimum was attained about May 26, when the moon reached its last quarter.

I had expected to find that variations of barometric pressure would exert an influence on the water-level in the tank A, and accordingly barograph records were taken on the farm simultaneously with the water-level records.

A comparison of the barograph records with the water-level records at first did not yield absolutely certain evidence of a connection between the two. During the June fortnight the barometric pressure remained very steady, and the barogram did not show any variation amounting to more than ·2 of an inch, and mere general inspection did not disclose any movements of the water-level curve that could be attributed to barometric change. During the May week, however, a greater variation of atmospheric pressure occurred, the barogram showing a gradual fall, extending through the greater part of the week, while the mean line of the water-level curve gradually rose during the same period. It was possible that this might have been a fortuitous coincidence, and it became obvious that much longer records would have to be secured if such a connection was to be established beyond doubt. Longer records would also be required to put on a more satisfactory basis the connection suggested above as existing between the great wave amplitudes and the lunar phases.

Messrs. Rayner and Roberts have from the beginning evinced a keen, intelligent interest in my investigations and a gratifying sympathy with the spirit of the inquiry. They have always been anxious to place all reasonable facilities and help at my disposal, but during these early years they were engaged in a very severe struggle with the difficulties of semi-arid farming. Their livelihood was dependent on their control of the water of these wells, and they were then unable to permit the uniformity of conditions necessary for the securing of a continuous record of two or three months. In particular they found it necessary from time to time to increase the yield of No. V. enormously by the insertion of a siphon. After the removal of the siphon the potential level recovered its normal position only after the lapse of days.

During the next two years I contented myself with occasional visits to the farm, during which I obtained a few short records of a day or two, which, with general observations, were sufficient to show that the fluctuations were apparently going on as before. I verified most of my earlier observations and examined the surface geology of the farm. The geological examination of the farm brought to light various facts which,

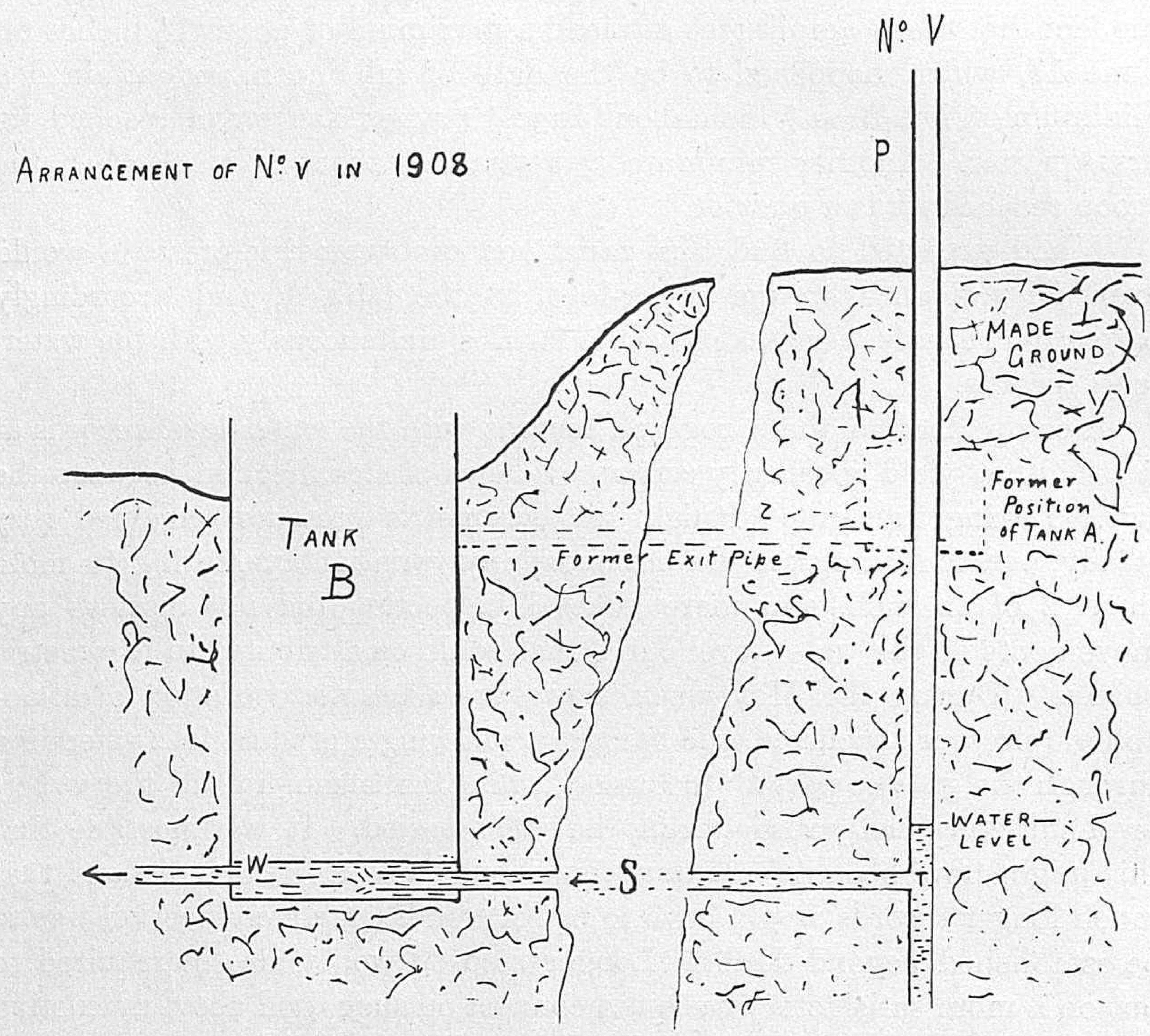


Fig. 5.

though interesting in other connections, did not seem to have any immediate bearing on the behaviour of the wells. The only observation that seemed to be worth mentioning in this connection was that the course of a dolerite dyke, prominently exposed on the mountains on the north boundary of the farm but hidden under the thick soil in the low ground, ran in a direction from a little east of north to a little west of south, in such a position as to indicate that its hidden portion probably passes between the two groups of boreholes.

During these years I observed no change in the temperatures of the waters.

Towards the end of 1907 Messrs. Rayner and Roberts informed me that they had made arrangements which would probably allow of a long record of several months' duration being taken during the early half of 1908.

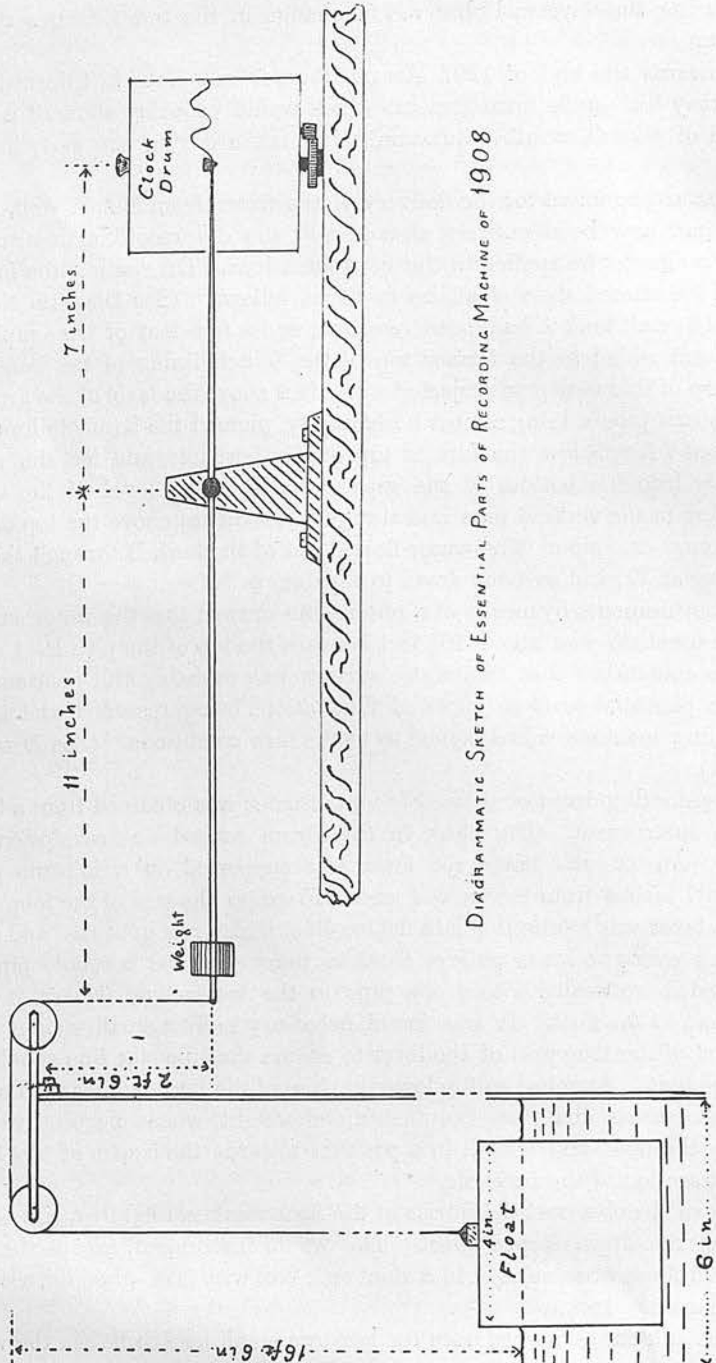
The arrangement for the delivery of the water from No. V. well, however, had now been entirely altered, and the old recording instrument could no longer be applied in the new conditions. On visiting the farm I found the altered state of affairs to be as follows. (See Diagram No. 5.) The old small tank A had been removed, and a few feet of iron piping *p* had been added to the former top of the 6-inch lining of the borehole. The top of this new pipe projected a few feet above the level of the ground. A new exit pipe *s*, lying almost horizontally, pierced the borehole lining at T about 7 feet below the former top of the borehole, and led the water directly into the bottom of the great tank B. The level of the water standing in the vertical pipe was always a few inches above the top of the horizontal exit pipe. The water flowed out of the tank B through the old opening at W, and so away down to the dam as before.

Measurements by means of a plumb-line showed that the water surface in the borehole was about 16½ feet beneath the top of the pipe P. I came to the conclusion that this water surface was probably still near enough to the potential level to allow of fluctuations being recorded, and a new recording machine was designed to fit the new conditions. (See Diagram No. 6.)

A recording drum of about 3·7 inch diameter was obtained from a barograph instrument. The clock in this drum caused one revolution per week An 18-inch brass rod lever was supported on a fulcrum pivot placed 7 inches from the writing end. Fixed to the end of the long arm of the lever was a length of silk fishing-line, which ran upwards, and after passing over two brass pulleys fixed to the top of the borehole pipe P, descended vertically within the pipe to the water-level, where it was attached to the float. It was found necessary to fix a small weight near the end of the long arm of the lever to ensure that the silk line should be always taut. Attached to the lower surface of the float by means of a few inches of brass chain was another metal weight, whose purpose was to steady the float and keep it in a position towards the centre of the horizontal section of the borehole.

It will be observed that a rise of the float was recorded by a rise of the pen on the drum record sheet. The whole instrument, except the silk line and float, was enclosed in a stout teak box with plate-glass lid, secured by means of a lock.

The silk cord emerged from the box by a small hole drilled in the wood



Diagrammatic Sketch of Essential Parts of Recording Machine of 1908

Fig. 6.

frame of the lid. The hole was so placed and of such a size as to ensure that the movements of the cord could never allow of any friction of the wood on the cord.

Mr. Cottell constructed the instrument, and as he was now stationed at Cradock he easily found opportunities of visiting the farm, setting up the instrument, and observing its behaviour.

As was to be expected, this second instrument did not work so smoothly as the simpler one of 1905. The increased friction due to the introduction of the pulleys partly accounted for this, and the weights on the lever and float required careful adjustment and balancing. The most advantageous balance of weights to use was found only after several unsuccessful trials. Mr. Cottell set up the instrument in the beginning of February, 1908, when I was also able to visit the farm for a day or two. The month of February was spent in attempts to reduce the friction and improve the working of the instrument. Mr. Cottell meanwhile sent me one by one such records as he obtained, together with his remarks on the action of the machine and his suggestions for mechanical improvement. By the beginning of March he reported to me that fairly satisfactory work could be expected.

From the beginning of March to the middle of June weekly record sheets were regularly obtained from the level recorder and from a barograph which I left on the farm. During all this period No. V. borehole was flowing uninterruptedly through the large tank into the dam, while the other boreholes on the farm were kept closed. Messrs. Rayner and Roberts assured me that during the four months they took the greatest care to ensure absolute uniformity of conditions as far as the human control of the boreholes was concerned. In the superior intelligence and trustworthiness of these gentlemen I have every reason to feel confidence. The records obtained, moreover, yield no evidence of any disturbance caused by other than natural causes.

Mr. Cottell undertook for me the general supervision of the instrument during the months February to June, paying frequent visits to the farm, and in his absence Mr. Rex Roberts undertook the weekly winding of the clocks and the weekly renewal of the record sheets. Mr. Cottell meanwhile posted weekly reports to me of the progress of the machine.

Mr. Roberts kept a careful record of the exact times at which each weekly record started and ended, and this, together with the lengths of the records afterwards measured by me, gave a means of estimating the average clock rates for each week.

The time as measured on Mr. Roberts's watch was compared with official railway time whenever any one on the farm made the journey to Mortimer Station or to Cradock. This communication with the railway takes place as a rule about once in every two days, but occasionally the flooding of the Fish River cuts off communication for four or five days. I have come to the conclusion that as a rule the railway time is known on the farm within 5 minutes, but that occasionally the error may have amounted to 20 minutes. Such occasional errors will, of course, be little appreciable on a record whose scale is such that 1 hour is represented by a length of curve amounting to $\frac{1}{20}$ inch.

Even after all efforts had been made to reduce the friction, the records gave evidence that much more friction remained than in the case of the 1905 machine. The upward and downward movement of the recording pen was often accomplished in a series of small jerks of about $\frac{1}{16}$ inch each, and occasionally of as much as $\frac{1}{4}$ inch. The machine was not sensitive to the larger gas-bubbles, and the general aspect of the line traced resembled the line traced by an ordinary barograph. There was little appearance of seich frill, and no hair lines due to impact of gas-bubbles on the float. Moreover, the machine did not seem to be capable of recording much of the semi-diurnal fluctuations when the amplitude was small—*i.e.*, about the times of moon's first and last quarters. The semi-diurnal fluctuations were clearly apparent, however, at all seasons of new and full moon, and thus one of the principal objects of taking the long record was successfully attained, as will be seen on Diagram No. 7, which is a photographic reduction of a tracing of all the records of these fifteen weeks.

See Plate IV

The fifteen record sheets were fitted together in order, and carefully traced by me as a continuous record. The junctions of the original week records are indicated by vertical dotted lines.

The observed data from which the clock rate for each week was calculated is shown in Schedule VI. Each record was divided up into 24-hour periods in accordance with the estimated average clock rate for its week, and the curved ordinates drawn to indicate the position of 9 a.m. for each day. The clock rate was always very near 12 inches per week.

As the recording lever had been divided by the fulcrum pivot in the proportion of 7 : 11, the amplitudes of the fluctuations on the record were less than the actual water fluctuations.

Theoretically if the machine had been free of friction and backlash, and if the float had been perfectly sensitive to small variations of water-level, the mechanical reduction of the amplitudes would have left the recorded amplitudes in a scale of $\frac{7}{11}$ of nature. As a matter of fact the imperfections of the machine conspired to reduce the amplitude scale still further to an unknown extent.

The barograph records were obtained on a time scale of about 11 inches per week, but this scale is sufficiently near the time scale of the level records for purposes of general comparison. Accordingly the barograph curve for each week was traced under the water-level curve. I have also indicated by insertion of the usual conventional signs the times of the moon's phases during the fifteen weeks.

A comparison of the barograph and water-level curves shows very clearly and satisfactorily that (if we ignore the semi-diurnal fluctuations) the mean water-level line in general rises when the barometric pressure falls, and vice versa. Thus in the beginning of the first week the barometer falls and the mean water-level rises, while about the end of the week the barometer rises and the mean water-level falls.

In the second week, during March 10, 11, and 12, the barometer falls and the mean water-level rises. On March 13 and 14 the barometer is higher and the mean water-level correspondingly lower. During March 15 and 16 the barometer is lower and the mean water-level higher. This connection between the two lines obviously continues throughout most of the fifteen weeks, any marked rise or fall of the barometer being accompanied by a corresponding fall or rise of the mean water-level. The week from June 8 to June 15 appears to me to be noteworthy as being the only week in which the above rule seems to fail. *

During June 10 and 11 the barometer shows a 24-hour period of high pressure without any obvious corresponding depression of the mean water-level, and on June 14 and 15 the mean water-level seems to be falling without any appearance of a barometric rise. It must be borne in mind that the barograph record is to be interpreted as indicating in a general way only the time of marked variations. I cannot regard its quantitative indications as being reliable within $\frac{1}{10}$ inch. This opinion is based on several weeks of daily comparisons which I have made of the indications of this barograph instrument with readings of a Kew mercurial barometer. I conclude, however, that the series of correspondences indicated on these fifteen weeks records is sufficiently notable and consistent to justify the belief that no theory of fortuitous coincidence will satisfactorily account for them, and that the records must be reasonably held to establish the general truth of the conclusion—

"*Increase of barometric pressure at Tarka Bridge is accompanied by a lowering of the general water-level in Borehole No. V., and vice versa.*"

An examination of the semi-diurnal fluctuations indicated on the water-level record shows that these fluctuations attain a maximum amplitude during the few days around each of the following dates: March 3, March 18, April 1, April 16, April 29, May 15, May 30, June 13, and that the amplitudes as recorded attain minimum and practically vanishing values in the neighbourhood of each of the following dates: March 8, March 25, April 8, April 23, May 8, May 22, June 6. See Plate IV

* Another exception occurs during the days May 22, 23, & 24.

Now the Lunar Calendar for the period under consideration is as follows:—

1908.

● March 2, 6.57 p.m. (Greenwich)	● April 1, 5.2 a.m.
☽ March 9, 9.42 p.m.	☽ April 8, 4.32 p.m.
○ March 18, 2.29 a.m.	○ April 16, 4.55 p.m.
☾ March 25, 0.22 p.m.	☾ April 23, 7.7 p.m.
● April 30, 3.33 p.m.	● May 30, 3.15 a.m.
☽ May 8, 11.23 a.m.	☽ June 7, 4.56 a.m.
○ May 16, 4.32 a.m.	○ June 14, 1.55 p.m.
☾ May 23, 0·17 a.m.	

The inference from this comparison seems now to be well established to the effect that—

"*The semi-diurnal fluctuations of level in Borehole No. V. attain a maximum amplitude about the times of New Moon and Full Moon, and attain a minimum amplitude about the times of the Moon's quadratures.*"

A thermograph record of fluctuations of atmospheric temperature on the farm was also obtained for the fifteen weeks, but a comparison of these temperature curves with the water-level curves revealed no connection of any kind as existing between the two, and nothing further need be said of them.

Comparison of Tarka Bridge Records with South African Coastal Tide Gauge Records.

The elementary observations already described had shown the behaviour of Borehole No. V. to have an intimate relationship to the moon's position, and it now appeared advisable to compare the Tarka Bridge records with the coastal tide gauge records.

See Curve A on Plate VI

For this purpose I selected the record for the fortnight June 4 to June 18 of 1905. My reasons for selecting this record were the following:—

(*a*) This record was taken on the better machine of 1905.

(*b*) It is a record on a large time scale over half an inch to the hour, and allows of greater accuracy in determining the times of successive high and low water in the semi-diurnal fluctuations.

(*c*) It is the record for a fortnight in which little change of barometric pressure was recorded at Tarka Bridge, and thus the principal disturbing factor local to Tarka Bridge is nearly eliminated.

The daily record sheets of Tarka Bridge were first examined and the curved ordinates of 9-inch radius inserted at the beginning and end of each record. Then the straight lengths of records were measured and the average clock rates for each day determined. The following list gives the time at which each record was put on and taken off, together with the length of record afterwards measured, and the deduced clock rate.

LIST OF JUNE, 1905, RECORDS AT TARKA BRIDGE.

Put on at—	Taken off at—	Length as Measured.	Clock Rate Calculated.
9.30 a.m., June 4	8.20 a.m., June 5	33·81 cm.	1.48 cm. per hour
8.22 ,, 5	8.0 ,, 6	33·50 ,,	1·42 ,,
8.2 ,, 6	8.0 ,, 7	35·18 ,,	1·47 ,,
8.2 ,, 7	8.0 ,, 8	35.28 ,,	1·47 ,,
8.2 ,, 8	8.0 ,, 9	35.18 ,,	1·47 ,,
8.2 ,, 9	8.0 ,, 10	35·21 ,,	1·47 ,,
8.2 ,, 10	9.29 ,, 11	35·10 ,,	1·44 ,,
9.30 ,, 11	8.0 ,, 12	34·41 ,,	1·53 ,,
8.0 ,, 12	8.0 ,, 13	34·53 ,,	1·44 ,,
8.0 ,, 13	8.15 ,, 14	35·05 ,,	1·45 ,,
8.15 ,, 14	8.15 ,, 15	34·98 ,,	1·46 ,,
8.15 ,, 15	8.15 ,, 16	35·28 ,,	1·47 ,,
8.15 ,, 16	8.15 ,, 17	34·90 ,,	1·45 ,,
8.15 ,, 17	8.30 ,, 18	35·06 ,,	1·45 ,,

The total length of record for the fortnight is 4·54 meters, and the average clock rate for the fortnight is 1·46 centimeters per hour.

The high-water and low-water points on each day record were then determined as carefully as possible, and the times at which these points were attained estimated by means of the various time scales indicated by the above clock rates.

At the same time the vertical heights of each of these maxima and minima were measured on the records, the upper edge of the record sheet being used as the datum line in these vertical measurements. The record sheets had been cut to fit accurately to the edges of the revolving drum.

The accompanying Schedule I. contains a list of times and heights of all these maximum and minimum points for the fortnight.

Tide gauges of the Légé type (see figure in G. H. Darwin's "Tides and Kindred Phenomena," page 11) have been established for a considerable number of years at Cape Town, Port Elizabeth, East London, and Durban. By the courtesy of the respective Government engineers in charge of these gauges, I have been enabled to examine the automatic records of these instruments for the months of May and June, 1905, and I have measured on these records the times and heights of all the high and low water turning-points for the periods, during which I obtained simultaneous records at Tarka Bridge in 1905.

On these tide-gauge records the vertical scale is 1 inch to the foot of water-level movement, and the horizontal or time scale is 1 inch to the hour. The records are in monthly lengths of 60 or 62 feet each.

On Schedules II., III., IV., and V. are presented the results of my measurements on these curves for the period extending from June 3 to June 20, and these figures may be compared with those obtained from my June records at Tarka Bridge, and shown on Schedule I. In this connection it may be well to remark that the determination of the exact time of any individual high or low water point on a tide curve is by no means easy.

The curve in the neighbourhood of a turning-point is as a rule so flat that two observers might differ in their estimate of the time in some cases by as much as a quarter of an hour or even more. G. H. Darwin remarks (on page 223 of the work already cited) that if the water rises only about a foot from low to high water in the course of five hours it is almost impossible to say with accuracy when it was highest, and two observers might differ by half an hour or even by an hour. It must be borne in mind that whereas the difference in the coastal tide curves amounted to several feet the fluctuations of level on the Tarka Bridge records amounted only to an inch or two. Hence, while I am of opinion that my determinations of the time of turning-points on the coastal tide curves are in most cases accurate within a quarter of an hour, I cannot hope to have attained similar accuracy in the determination of the times of high and low water on the Tarka Bridge curve. Any individual determination may easily be in error by half an hour, and occasionally by an hour or even more. These errors, however, tend to disappear in the average periods calculated over the entire fortnight.

See Plate V.

On Diagram No. 8 the data contained in the Schedules I., II., III., IV., and V. have been plotted in parallel lines to the same time scale, and this diagram exhibits the general similarity of the Tarka Bridge curve to the coastal tide curves.

The principal dissimilarities would appear to be—

(1) The scale of the semidiurnal amplitudes. These amplitudes in the Tarka Bridge curve range through a couple of inches, while in the coastal curves they range through several feet.

(2) The amplitude of the diurnal inequality seems to be relatively greater in the Tarka Bridge curve than in the coastal curves.

The following averages are easily calculated from the scheduled data.

From Schedule I.—

From first L.W. at 11.55 a.m., June 4,
to last H.W. at 8.0 a.m., June 18

is a period of 332 hrs. 5 min., and comprises 26½ wave-lengths. Average wave period at Tarka Bridge: 12 hrs. 32 min.

From Schedule II.—

From L.W. at 11 a.m., June 4,
to H.W. at 5.5 a.m., June 18

is a period of 330 hrs. 5 min., comprising 26½ wave-lengths on the tide curve. Average wave period at East London: 12 hrs. 28 min.

From Schedule IV.—

From L.W. at 10.31 a.m., June 4,
to H.W. at 4.25 a.m., June 18

is a period of 329 hrs. 54 min., comprising 26½ wave-lengths on the tide curve. Average wave period at Durban: 12 hrs. 25 min.

From Schedule V.—

From L.W. at 11.15 a.m., June 4,
to H.W. at 5.5 a.m., June 18

is a period of 329 hrs. 50 min., comprising 26½ wave-lengths on the tide curve. Average wave period at Cape Town: 12 hrs. 25 min.

The nearest point of the South African coast to Tarka Bridge is on the northern shore of Algoa Bay, and the distance is about 100 miles measured on a straight line. Port Elizabeth is about 114 miles from Tarka Bridge, and the distance between East London and Tarka Bridge is about 136 miles.

The East London tide gauge is the nearest one which was giving reliable records during the early half of June, 1905.

If we tentatively correlate the East London record with the Tarka Bridge record in such a way as to regard the East London high water of 4.55 a.m., June 4, as corresponding to the Tarka Bridge high water of 5.45 p.m. on the same day, and correlate all the remaining high waters consistently with this first assumption, we find that for the period June 4 to June 18 the average lag of Tarka Bridge high waters behind East London high waters is 14 hrs. 27 min., while a similar treatment of the low waters of the same period gives an average lag of Tarka Bridge low water behind East London low water amounting to 14 hrs. 51 min.

The difference in these two averages seems too great to be due to observational errors and raises a strong suspicion that the Tarka Bridge lag at low water is really greater than the lag at high water. Now if this is really so the curve records at Tarka Bridge will be slightly steeper when rising from low to high water than when falling from high to low water, and this is a matter which is easily investigated, the necessary data being contained in Schedule I.

If we take the difference of each time of low water as given in Schedule I. and the time of the succeeding high water we find that the average time interval between low water and the succeeding high water at Tarka Bridge is 5 hrs. 53 min. Then taking the time interval between each high water and the succeeding low water, we find the average interval to be 6 hrs. 35 min.

A comparative inspection of two sides of each individual wave represented in these lists of half-wave intervals makes the conviction stronger that we are here dealing with no imaginary phenomenon, for in the list of 27 ascending half-waves and 26 descending half-waves we see that in 20 individual cases the ascending half has a shorter interval than the descending half immediately following. In two cases the intervals are practically equal, and in the remaining four cases the ascending interval is greater than the descending interval. In each of the anomalous cases an inspection of the adjacent intervals strongly suggests that an unusually large error has occurred in the determination of the culminating point of the curve, and this would entirely account for the apparent anomalies.

I conclude that the inference is a fair one that in general a shorter time elapses between low water and high water than between high water and the succeeding low water of the well. In other words, at Tarka Bridge the water rises more quickly than it falls, and thus the rise and fall of this well water is to that extent analogous to the tidal fluctuations in an estuary or tidal river rather than to the tides in the open ocean. ~~As will be seen later,~~ This result is important as having a bearing on the questions arising as to the nature of natural mechanism by which the tidal influence is transmitted from the ocean to the well, if such a theory of the origin of Tarka Bridge tides be admissible.

The correlation of a particular high water on the coastal tide curve with a particular high water on the Tarka Bridge curve cannot be regarded as satisfactorily accomplished, and although in the foregoing discussion I have stated the lag of Tarka Bridge behind East London to be for high waters 14 hrs. 27 min., all I have really proved is that the lag is 14 hrs. 27 min. $+ p \,.\, 12\frac{1}{2}$ hrs., in which expression p is a constant integer of undetermined positive or negative value.

It is indeed open to question whether such attempts at correlation are not from their very nature wholly illusory, since there is as yet no clear evidence that the Tarka Bridge tide is produced as a consequential effect of oceanic tides. The connection between the two may be merely that of a common astronomical cause.

Determination of the various Harmonic Periods in the Tarka Bridge Curve.

The advisability of subjecting the Tarka Bridge records to a harmonic analysis occurred to my mind at an early stage of my investigations.

On inquiring into the methods of harmonic analysis commonly adopted in the study of marine tide curves I found that these methods are based on assumptions that the curves consist of harmonic components the periods of which are known or assumed from astronomically observed data. As my object was to prove or disprove a definite connection between the harmonic components of the curve and astronomical data, I concluded that the above-mentioned methods were inapplicable for my purpose. Lately there came under my notice a method invented by Prof. Chrystal for the determination of the periods of the harmonic components of limnograms or seiche curves. This is a method which is based on no assumptions as to the nature of the causes producing the movements indicated on the curves, and a careful consideration of the mathematical theory of the method convinced me that it could be applied with perfect confidence to the Tarka Bridge records. Chrystal speaks of it as the "Method of Residuation," and its theory and mode of application is described fully in Trans. Roy. Soc., Edinburgh, vol. xlv., part 2, pp. 385–7. As this reference is not easily accessible to most South African readers, and as the method is little known, I venture to quote it in full in an Appendix.

From the form in which Chrystal gives his mathematical theory it may not be immediately obvious that the method is applicable to the Tarka Bridge curves, and the following explanation may perhaps be necessary.

Fourier has shown that any finite periodic function of a variable can be expressed as a series of terms, each of which is a simple harmonic function of this variable.

Thus if y be any periodic function of the time, y can by suitably choosing the constants A, M_1, M_2, &c., e_1, e_2, &c., be expressed by a series of terms as follows—

$$y = A + M_1 \sin (kt + e_1) + M_2 \sin (2kt + e_2) + \&c.$$

Without making any further assumptions as to the nature of the function this equation can be transformed into the form—

$$y = A_1 \sin \frac{2\pi}{T_1} (t - a_1) + A_2 \sin \frac{2\pi}{T_2} (t - a_2) + \&c.,$$

which is the equation for the curve assumed by Chrystal in his mathematical theory of the Residuation Process.

The method therefore is applicable to every curve which is the expression of a finite periodic function of a variable, and hence obviously to the Tarka Bridge records.

The residuation method is essentially a means of eliminating from a curve one by one its various harmonic components. My actual mode of working may be described as follows.

The curve to be residuated is traced on to another sheet, and then a second tracing of the same curve is superimposed on the same sheet but with the original curve moved along the time axis through a distance corresponding to half the period of the harmonic to be eliminated. Then a line is drawn so that it is everywhere half-way between the two tracings, and the intermediate curve thus obtained is the first residuum. It contains all the harmonic components of the original except one, with their periods unaltered but with their amplitudes considerably reduced.

I have found the process somewhat laborious in practice, but the results are much better than I had expected.

Residuation Method applied to Tarka Bridge Curve.

The same reasons already mentioned as leading to the selection of the record June 4 to June 18, 1905, for purposes of comparison with marine tide records held good in the consideration as to which of the Tarka Bridge records was most suitable for residuation analysis, and accordingly the record June 4 to June 18 was again selected.

The daily records for that period were fitted together and traced as a continuous curve on tracing cloth. This continuous record was almost 5 meters long. The tracing was made in the form of a smooth curve carried through a mean position in the seiche frills of the original. (This is the curve A at the top of the photo-reduced Diagram 9.)

See Plate VI

The times of the first and last turning-points were determined as follows:—

First L.W. at 11.55 a.m., June 4,
Last H.W. at 8·0 a.m., June 18,

giving a period of 332 hrs. 5 min., comprising 53 half-waves of the dominant harmonic, or—

$$\frac{53T_A}{2} = 332 \text{ hrs. } 5 \text{ min.},$$

from which we deduce 12 hrs. 32 min. as a first approximation for the value of T_A.

The curve A was now residuated with respect to $T = 12$ hrs. 32 min., and yielded a curve C. This curve C was apparently composed of several harmonic components whose periods were very similar to each other, but one of these components had a considerably greater amplitude than the others. Attention was confined to this dominant component. By placing the eye near the level of the cloth at one end and glancing along the foreshortened curve, I was able to count 14 crests and 14 troughs in the curve.

I determined as nearly as possible the time positions of two or three of these turning-points near the beginning and end of the curve, and obtained the following results :—

(i) From 1st L.W. to 14th L.W. a period of 311 hrs. 59 min.

$$\therefore \quad T_C = \frac{311 \text{ hrs. } 59 \text{ min.}}{13} = 24 \text{ hrs.}$$

Owing to flatness of curvature and proximity to the beginning of the curve, the 1st H.W. is a badly defined point.

(ii) From 2nd H.W. to 14th H.W. is a period of 286 hrs. 17 min.

$$\therefore \quad T_C = \frac{286 \text{ hrs. } 17 \text{ min.}}{12} = 23 \text{ hrs. } 51\tfrac{1}{2} \text{ min.}$$

Taking the mean of these two values of T_C, we get for the period of the dominant harmonic of curve C as a first approximation the value 23 hrs. 56 min.

Returning to the original curve A and residuating it with respect to $T = 23$ hrs. 56 min., a second residual curve D was obtained. This curve D of course strongly resembled curve A, and, like it, showed 26 crests and 26 troughs. That curve D contained more than one harmonic component, however, was obvious from the fact that the amplitudes were smaller near the middle of the fortnight than near the beginning and end.

The times of the first and last H.W. and L.W. were determined, giving the following results :—

(i) From 1st L.W. to 26th L.W. was a period of 311 hrs. 41 min.

$$\therefore \quad T_D = \frac{311 \text{ hrs. } 41 \text{ min.}}{25} = 12 \text{ hrs. } 24 \text{ min.}$$

(ii) From 2nd L.W. to 26th L.W. was a period of 312 hrs. 0 min.

$$\therefore \quad T_D = \frac{312 \text{ hrs.}}{25} = 12 \text{ hrs. } 29 \text{ min.}$$

(iii) From 2nd L.W. to 26th L.W. was a period of 299 hrs. 33 min.

$$\therefore \quad T_D = \frac{299 \text{ hrs. } 33 \text{ min.}}{24} = 12 \text{ hrs. } 29 \text{ min.}$$

Averaging these three values, we get as a first approximation—

$$T_D = 12 \text{ hrs. } 27 \text{ min.}$$

This is of course really a second approximation to the value of T_A.

Again, returning to curve A and residuating it with respect to $T = 12$ hrs. 27 min., I obtained a residual curve F almost identical with the curve C already obtained.

On curve F the following results were obtained :—

(i) From 1st L.W. to 14th L.W. was a period of 311 hrs. 34 min.

$$\therefore \quad T_F = \frac{311 \text{ hrs. } 34 \text{ min.}}{13} = 23 \text{ hrs. } 58 \text{ min.}$$

(ii) From 2nd H.W. to 14th H.W. was a period of 286 hrs. 52 min.

$$\therefore \quad T_F = \frac{286 \text{ hrs. } 52 \text{ min.}}{12} = 23 \text{ hrs. } 54 \text{ min.}$$

(iii) From 3rd H.W. to 14th H.W. was a period of 263 hrs. 50 min.

$$\therefore \quad T_F = \frac{263 \text{ hrs. } 50 \text{ min.}}{11} = 23 \text{ hrs. } 59 \text{ min.}$$

Averaging these three values we obtain—

$$T_F = 23 \text{ hrs. } 57 \text{ min.}$$

This is of course really a second approximation to the value of T_C, and it thus appears that the limits of working accuracy were attained in the first residuation.

The curve F was next residuated with respect to its dominant harmonic period $T = 23$ hrs. 57 min., and yielded a curve H. No regular periodic undulation can be discerned on the curve H. By placing the eye near the level of the tracing cloth and glancing along the foreshortened curve, one distinguishes a fluctuation of irregular periodicity. Three crests and two troughs can be seen in the entire curve. As the curve is very flat and the undulation very long the positions of some of these turning-points can only be estimated with a probable error that might amount to nearly a day, while in the case of others the error might not exceed 6 hours.

The curve begins with a slight upward movement reaching a maximum about the afternoon of June 5 or the morning of June 6. Then it slowly declines to a minimum about the afternoon of June 7.

Next it slowly rises, attaining a maximum somewhere in the morning of June 9, after which it falls to a minimum in the morning of June 13.

Following this comes the most noteworthy movement of the curve. It rises more rapidly than usual, attaining the highest of all the maxima in the afternoon June 14, and remains very high from that point to the end in June 17, though a slight downward movement marks the last two days of its course. During June 14, 15, and 16 the line is continuously higher than in any other part of the fortnight.

It has been already remarked that the barograph curves taken on the farm during this fortnight indicated comparatively little variation of atmospheric pressure during the period.

A close examination shows that the barogram for the fortnight contains two principal maxima and three minima.

A slight downward movement begins in the morning of June 5, and a minimum is reached about 4 p.m. on June 6, after which a slight upward movement culminates in a maximum about 11 a.m. on June 7. Then a slight downward movement follows, reaching a minimum about noon on June 9, after which a gradual upward movement carries the curve to a maximum at 9 a.m., June 13.

Now follows the greatest movement of the fortnight. The line falls about ·2 of an inch between the last-mentioned time and the minimum at 6 p.m. of June 14. From that date on to the morning of June 18 there is little movement.

A comparison of the curve H with the barograph curve makes it apparent that the former is substantially the vertical inversion of the latter, the maxima of the one corresponding in time to the minima of the other.

Let us return now to the consideration of the residual curve D. Although this curve is obviously more uniformly regular than curve A, nevertheless it is apparent that it contains more than one harmonic component. This is indicated by the varying amplitudes of its waves. The obvious interpretation is that it consists of two harmonics whose periods do not differ much from each other—both being approximately semi-diurnal periods. In the middle of the fortnight when the amplitudes of curve D are smallest a minimum point of one of the harmonics synchronises with a maximum point of the other harmonic component. At the beginning and end of the fortnight when the amplitudes are greatest the maxima of the one harmonic tend to coincide with the maxima of the other harmonic. Now in the middle of the fortnight the moon was in its first quarter. One of the general results already obtained from the very long record of 1908 was the fact that amplitudes of the water-level curve are regularly at a maximum at the time of full and new moon and at a minimum during the moon's first and third quarters. This fact implies that the two principal harmonics in the curve have their respective maxima synchronising twice in a lunar month or once in 14 days. It follows from this that 14 days must be the least common multiple of the periods of the two harmonics.

The measurements already made on curve D have shown that its dominant harmonic period must be in the neighbourhood of 12 hrs. 27 min., or nearly $\frac{1}{27}$ of a fortnight. It now appears probable that it is exactly $\frac{1}{27}$ of a fortnight, or 12 hrs. 26 min., and that the remaining harmonic must either have a period of $\frac{1}{28}$ of a fortnight, or 12 hours, or alternatively $\frac{1}{26}$ of a fortnight.

In order to test the reliability of this last probable hypothesis in a practical way, the curve D was residuated with respect to T = 12 hrs., and in this way was obtained a residual curve F. This curve was composed of waves of small amplitude amounting on the average to about ·14 of an inch, and it was evident that in this residuation I was approaching the lower limits of accurate working. I determined 24 H.W. points and 25 L.W. points in the curve, and measured each wave interval both from L.W. to H.W. and also from H.W. to H.W.

In this way I found the average distance from H.W. to H.W. to be 18·16 cm., and the average wave-length as measured from L.W. to L.W. to be 18·15 cm.

Taking the average clock rate as 41·06 minutes per centimeter, and converting these averages into time, we get the values: 12 hrs. 25½ min., and 12 hrs. 25 min. This result, though not obtained by strict adherence to the Residuation process, at least shows that curve C is plausibly divisible into two harmonic components having periods of 12 hrs. 26 min. and 12 hrs. respectively. The possibility of its being equally divisible into another pair of components is not quite excluded.

Summary of the Results of the foregoing Analysis.

The record curve obtained at Tarka Bridge during the period June 4 to June 18, 1905, has been dissected into two curves having as their main harmonic periods

12 hrs. 27 min.,

and 23 hrs. 57 min. respectively,

together with an anharmonic residuum which appears to correspond substantially to a vertical inversion of the barogram for the fortnight obtained on the farm.

Further the curve of 12 hrs 27 min. periodicity has been shown to be plausibly divisible into two harmonic components having periods of 12 hrs. 26 min. and 12 hrs. respectively.

These results may be compared with the following principal harmonic components of marine tides (see "The Tides," by G. Darwin, p. 182).

I. Principal lunar semi-diurnal tide: Period, 12 hrs. 25 min. $14\frac{1}{8}$ sec.
II. Principal solar semi-diurnal tide: Period, 12 hours exactly.
III. Three diurnal tides: Periods, 23 hrs. 56 min.; 24 hrs. 4 min.; 25 hrs. 49 min. $9\frac{1}{2}$ sec.

The lunar periodicity of the well at least may be regarded as established.

Tidal Movements of Wells No. III. and No. VI.

In January, 1912, I spent a week on the farm and devoted my attention mainly to preliminary observations on Nos. III. and VI. I ascertained that Nos. II., III., and VI. had been continuously open for many months.

I closed Nos. III. and VI. by means of wooden plugs, through which projected tightly fitting lead tubes. These lead tubes projected vertically about 7 or 8 feet above the plugs, and were surmounted by lengths of glass tubing in which one could observe the rise of the water-columns to their potential levels. Arbitrary scales were attached to the glass tubes by means of which numerical values could be assigned to the levels of the water-columns in the tubes.

During the first few days the levels rose as the wells recovered the loss of potential due to their having been open so long. Then readings were taken during January 20 and 21.

The results plotted on squared paper indicate that the mean level continued to rise during these days, and that the process of recovery of lost potential was still continuously in progress. The theoretical "cone of depression" was still filling up around the well.

The curves, however, show in addition a distinct semi-diurnal fluctuation similar to that of No. V. in previous records. On January 21, between 6 and 7 p.m., No. VI. was opened for 14 minutes, and the effect of this is obvious on the No. III. record. This proves a close underground communication between No. III. and No. VI.

I determined the time interval for transmission of change of pressure between these two wells as 50 seconds.

Over No. II. there has been erected a windmill and pump, by means of which the water is raised and piped for domestic use into a tank on the roof of Mr. Roberts's house.

On January 22, about 6 p.m., this mill was put into action, vigorous pumping from No. II. began, and this at once began to effect a depression of the level on No. VI.

Thus it appears that Nos. II., III., and VI. have a more or less direct underground connection.

Arrangements are now being made to have an improved recording machine placed on No. VI. for the purpose of obtaining longer records. The short records already obtained suggest that the H.W. of No. VI. is not synchronous with that of No. V., and that there may be a time difference between the two amounting to several hours. Longer records, however, are necessary for the purpose of proving or disproving this want of synchronism. Various reports have reached me alleging the existence of fluctuations in several other sulphurous wells in the eastern part of Cape Colony, but I have not as yet had any opportunity of verifying them.

Analyses of the Water and Gas of the Tarka Bridge Wells.

A sample of the water which I took from No. V. in 1905 was analysed by Mr. John D. Rose, of the Government Analytical Laboratory. The following are Mr. Rose's analytical results :—

	In Grains per Gallon.
Total solids at 180° C.	38·80
Silica	1·78
Oxide of iron and alumina	0·06
Lime	3·82
Magnesia	1·23
Alkalies calculated as Na	13·67
Chlorine	13·79
Sulphuric oxide	0·33
Combined carbonic dioxide	5·33

"All the lime and magnesia are present as carbonates, with a trace of calcium sulphate, while sodium chloride appears to be the chief mineral constituent. The spectroscope reveals the presence of a very minute quantity of lithium. Potassium is also present in small quantities as shown by the flame reaction."

Mr. Rose did not attempt to estimate the dissolved gases, but part at least of the sulphuric oxide figuring in his analysis is probably due to oxidation of the sulphuretted hydrogen contained in the water as it issues from the well. The odour leaves the water after it has stood exposed to the air for a few hours. I have found that the water of all the Tarka Bridge wells tested on the spot immediately after collection, yields with cadmium chloride solution a considerable precipitate of yellow cadmium sulphide.

I have made various rough analyses of the gases escaping from the wells, showing that methane is the principal constituent in each case, and that oxygen and carbon dioxide are absent. A gas sample which I collected in January, 1912, from No. III. was analysed by Mr. James Moir, M.A., D.Sc., F.C.S., at Johannesburg, and yielded him the following results :—

Methane	94·0 per cent.
Hydrogen..	2·7 ,,
Nitrogen and other unabsorbables	3·2 ,,
Oxygen and carbon dioxide—traces, together less than	0·1 ,,

I have observed various loose stones (some of them apparently showing signs of Bushman work) which seem to have been lying for a long time in the water escaping from the natural springs beside No. V. These stones are covered with a smooth siliceous enamel, and some of them with a smooth coating of iron pyrites. During the progress of a little excavation around the spring I also observed that the shales exposed were highly impregnated with small crystals of iron pyrites. The inference seems unavoidable that the sulphurous water deposits the iron pyrites from solution or by reaction of the sulphuretted hydrogen on the ferruginous constituents in the shales.

The bacterium already alluded to is present in all the five boreholes and in the natural springs associated with them. It appears as white or pink feathery loose tufts, which from time to time are emitted in the escaping water. On exposure to the air and sunlight these tufts soon become dark in colour. I submitted specimens to my colleague Professor Pearson, South African College, and he informs me that it is a sulphur bacterium which flourishes in water impregnated with sulphuretted hydrogen. I have observed similar tufts issuing with the water of the sulphurous springs of Cradock and Aliwal North.

As to the quantity of gas delivered by the boreholes I have no very definite data. Nos. III. and VI. seem to be the holes which are most prolific in gas, but the rate of gas discharge obviously varies much in all the wells. On several occasions I inserted a 6-inch funnel in inverted position into the borehole, and so collected most of the escaping gas as it issued from the thin end of the funnel. The gas was led through a pipe into a 3-litre bottle, from which it easily displaced the contained water. I noted the time taken to fill the bottle with gas. The results indicated that No. III. and No. VI. each delivered from 3 to 4 litres of gas per minute. Messrs. Rayner and Roberts state that the rate of gas delivery as estimated roughly by the apparent trouble of the water surface is considerably affected by changes of weather, and that they obtain predictions of approaching storms in this way. It seems probable that great falls in the barometric pressure would increase the rate of gas delivery, by allowing the expansion and partial escape of quantities of gas partially imprisoned in underground branch fissures with low outlets communicating with the main fissures forming the water pathways.

Summary.

Observations begun in 1905 and carried on at intervals until the present year on a group of wells on a farm at Tarka Bridge, Cradock District, are described in detail.

The wells have not been bored very deep, the deepest being 225 feet, but it is obvious that the bores connect with deeply extending fissures, as the waters issue at temperatures of about 80° accompanied by large quantities of natural inflammable gas (methane), while sulphuretted hydrogen is present in notable quantities in solution in the water. The wells are 2,700 feet above sea-level and over 100 miles from the coast.

Measurements of the pressure at which the water issues show a remarkable fluctuation, in some respects analogous to the tidal fluctuations of the sea.

A series of direct measurements covering several days established the fact that there was a real fluctuation both in the amount of water discharged and in the well-pressure. Continuous records were then obtained over longer periods by means of clock-driven, self-recording apparatus in order to study the precise nature of the fluctuations.

The longest continuous record obtained extends over a period of fifteen weeks. This graphical record shows that the semi-diurnal fluctuations attain a maximum amplitude at fortnightly intervals at times corresponding to the times of new moon and full moon throughout the fifteen-weeks' period.

This record further demonstrates the fact that the mean daily water pressure rises with each fall of barometric pressure, and falls with each rise in barometric pressure as recorded concurrently at the farm by means of a barograph instrument. The time scale on this fifteen-week record is about 11 inches per week.

Records obtained for shorter periods on a time scale of $13\frac{1}{2}$ inches per day were found to be much more suited for detailed critical examination and analysis. In particular, the record for a certain fortnight during which the barometric pressure was very steady (and consequently its interfering effect almost negligible) was selected. The times of all the turning-points were carefully determined in terms of South African official time. The heights of all the turning-points of the curve were also determined in inches.

Similarly, the co-ordinates of all the turning-points for that fortnight were determined on the tide gauge records of the South African ports of Cape Town, Port Elizabeth, East London, and Durban.

For general comparison all these measured data were plotted in parallel lines on the same time scale, and the general resemblance of the well curve to the curves of the coastal tide records demonstrated.

The original Tarka Bridge record for the fortnight was then subjected to a process of harmonic analysis for the purpose of determining the periods of the principal harmonic components of the curve. The particular method used was described by Chrystal as the method of "Residuation" (Trans. Roy. Soc., Edinburgh, vol. xlv., part 2, pp. 385–7). This method is applicable to comparatively short curves and involves no assumptions as to periods or the causes operating to produce the curve. One by one the various simple harmonic components are disentangled from the compound curve with their periods unaltered but with their amplitudes considerably reduced.

In this way components were isolated from the Tarka Bridge curve having the following wave periods:—

1. 12 hrs. 27 min. [probably divisible into 12 hrs. 26 min. and 12 hrs. 0 min.]
2. 23 hrs. 57 min.
3. An anharmonic residuum which appeared to be the vertical inversion of the barograph curve for the fortnight.

Component No. 2 was obviously not a simple harmonic function. It ~~was apparently~~ seemed to me to be composed of several harmonics of approximately diurnal period, but on the scale on which the analysis was being conducted the practical limit of the method had been reached. Accordingly no finer dissection was attempted.

The above results may be compared with the well-known principal harmonic components of marine tides.

1. Principal Lunar semi-diurnal tide—period 12 hrs. 25 min. $14\frac{1}{6}$ sec.
2. Principal Solar semi-diurnal tide—period 12 hrs.
3. Three diurnal tides with periods—23 hrs. 56 min.; 24 hrs. 4 min.; 25 hrs. 40 min. $9\frac{1}{2}$ sec.

General Remarks.

The foregoing results seem to establish beyond question that the fluctuations in these wells are to be attributed directly or indirectly to extra-terrestrial causes, but the precise nature of the connection is not by any means clear.

The wells are situated over 160 miles from the coast, at an altitude of over 2,700 feet above sea-level. High water at Tarka Bridge occurs about $14\frac{1}{2}$ hours after high water at East London, while the lag in the case of low water is nearly 15 hours.

The principal conceivable theories to account for the phenomena would appear to group themselves in three classes:—

A. Theories depending on the direct gravitative influence of the sun and moon on the land or the underground water.

B. Theories depending on the action of the marine tides on the coast loading and distorting the land.

C. Theories depending on the action of marine tides in periodically reducing the freedom of outflow of underground water through submarine springs.

No attempt is at present made to state or discuss these theories. It is felt that a satisfactory theory can be arrived at only by the co-operative discussion of the subject by astronomers, geologists, and hydraulicians.

Tidal wells are known in many parts of the world, but practically all are within 3 or 4 miles of the seashore and at no considerable altitude. The records from the Orisino Bore in Australia do not show the periodicity of marine tides.

One case is reported at Lille in France, 40 miles from the coast, but at no great height above sea-level. The evidence supporting the tidal claim of this well is not quite ~~far from~~ satisfactory.

It is believed that there is no other record of an inland well showing fluctuations of true lunar periodicity.

SCHEDULE I.

Measurements of Turning-points on Water-level Records on Borehole No. V. "Tarka Bridge," June, 1905.

High Water.			Low Water.		
Date.	S.A. Time.	Height.	S.A. Time.	Height.	Date.
		Inches.		Inches.	
			11.55 a.m.	2·10	June 4
June 4	5.45 p.m.	3·01	1.10 a.m.	1·49	,, 5
,, 5	6.55 a.m.	3·12	1.10 p.m.	2·25	
	6.0 p.m.	3·19	1·10 a.m.	2·00	,, 6
,, 6	7.25 a.m.	3·36	2.15 p.m.	2·39	
	7.25 p.m.	3·03	2.40 a.m.	1·94	,, 7
,, 7	9.0 a.m.	3·10	3.0 p.m.	2·34	
	8.20 p.m.	2·96	3.25 a.m.	1.99	,, 8
,, 8	11.2 a.m.	2·83	4.56 p.m.	2·35	
	10.32 p.m.	3·02	4.10 a.m.	2·38	,, 9
,, 9	10.50 a.m.	3·16	5.40 p.m.	2·29	
	11.45 p.m.	2·78	5.50 a.m.	2·12	,, 10
,, 10	12.56 p.m.	3·13	8.22 p.m.	2·21	
,, 11	2.25 a.m.	2·88	8.51 a.m.	2·22	,, 11
	1.25 p.m.	3·18	8.0 p.m.	2·13	
,, 12	2.30 a.m.	3·03	8.40 a.m.	2·19	,, 12
	2.15 p.m.	3·01	9.10 p.m.	1·94	
,, 13	3.30 a.m.	2·93	9.20 a.m.	2·00	,, 13
	3.10 p.m.	3·06	10.0 p.m.	1·91	
,, 14	4.10 a.m.	3·38	10.56 a.m.	2·50	,, 14
	4.18 p.m.	3·58	12.2 a.m.	1·96	,, 15
,, 15	5.37 a.m.	3·27	11.10 a.m.	2·21	
	4.30 p.m.	3·22	11.40 p.m.	1·78	
,, 16	7.0 a.m.	3·50	11.52 a.m.	2·52	,, 16
	5.5 p.m.	3·35	12.3 a.m.	1·63	,, 17
,, 17	6.43 a.m.	3·06	12.10 p.m.	2·11	
	5.45 p.m.	3·21	12.45 a.m.	1·70	,, 18
,, 18	8.0 a.m.	3·27			

Heights measured from Arbitrary Datum Line being upper edge of Record Sheets which were cut to fit accurately to the edges of the revolving drum.

Times are expressed in South African Civil Time (2 hours E. of Greenwich).

SCHEDULE II.

Data Measured on East London Tide Gauge Record, June, 1905.

High Water.			Low Water.		
Date.	S.A. Time.	Height.	S.A. Time.	Height.	Date.
		Feet.		Feet.	
June 3	4.25 a.m.	5·20	10.35 a.m.	1·08	June 3
	4.50 p.m.	5·15	10.35 p.m.	1·35	
,, 4	4.55 a.m.	5·28	11.0 a.m.	1·17	,, 4
	5.25 p.m.	5·41	11.15 p.m.	1·50	
,, 5	5.35 a.m.	5·43	11.35 a.m.	1·35	,, 5
	6.5 p.m.	5·52	11.45 p.m.	2·06	
,, 6	6.20 a.m.	6·00	12.15 p.m.	2·21	,, 6
	6.25 p.m.	5·98	12.30 a.m.	2·24	,, 7
,, 7	6.45 a.m.	5·43	1.0 p.m.	1·75	
	7.20 p.m.	5·30	1.15 a.m.	2·22	,, 8
,, 8	7.35 a.m.	5·17	1.35 p.m.	2·03	
	8.0 p.m.	5·10	2.20 a.m.	2·41	,, 9
,, 9	8.40 a.m.	4·86	2.35 p.m.	2·33	
	9.15 p.m.	4·89	3.35 a.m.	2·49	,, 10
,, 10	9.50 a.m.	4·47	3.40 p.m.	2·44	
	10.30 p.m.	5·00	4.50 a.m.	2·61	,, 11
,, 11	11.15 a.m.	4·61	5.10 p.m.	2·72	
	11.55 p.m.	5·28	6.30 a.m.	2·65	,, 12
,, 12	12.40 p.m.	4·94	6.20 p.m.	2·80	
,, 13	12.55 a.m.	5·52	7.15 a.m.	2·27	,, 13
	1.35 p.m.	5·12	7.40 p.m.	2·21	
,, 14	1.45 a.m.	5·48	8.10 a.m.	1·60	,, 14
	2.35 p.m.	4·98	8.25 p.m.	1·60	
,, 15	2.45 a.m.	5·57	9.0 a.m.	1·38	,, 15
	3.15 p.m.	5·30	9.10 p.m.	1·47	
,, 16	3.25 a.m.	5·65	9.40 a.m.	1·23	,, 16
	4.20 p.m.	5·63	9.50 p.m.	1·51	
,, 17	4.25 a.m.	6·00	10.30 a.m.	1·59	,, 17
	5.0 p.m.	6·31	10.40 p.m.	2·01	
,, 18	5.5 a.m.	6·41	11.5 a.m.	1·66	,, 18
	5.25 p.m.	6·12	11.20 p.m.	1·79	
,, 19	5.45 a.m.	6·07	11.45 a.m.	1·78	,, 19
	6.10 p.m.	6·06	12.10 a.m.	1·83	,, 20
,, 20	6.15 a.m.	5·39	12.25 p.m.	1·24	
	6.45 p.m.	5·08	12.40 a.m.	1·52	,, 21
,, 21	6.55 a.m.	4·72	12.55 p.m.	1·25	
	7.25 p.m.	4·75			

Heights measured from East London Harbour Datum Mark.
Times are all South African Civil Times (2 hours E. of Greenwich).

SCHEDULE III.

Data Measured on Port Elizabeth Tide Gauge Record, June, 1905.

High Water.			Low Water.		
Date.	S.A. Time.	Height.	S.A. Time.	Height.	Date.
		Feet.		Feet.	
June 4	4.36 a.m.	4·81	10.26 a.m.	−0·33	June 4
			*7.47 p.m.	−0·10	June 14
June 15	*2.20 a.m.	4·88	8.34 a.m.	−0·04	,, 15
	2.43 p.m.	4·74	8.27 p.m.	+0·11	
,, 16	2.55 a.m.	5·54	9.4 a.m.	0·41	,, 16
	*3.21 p.m.	5·75	9.3 p.m.	0·88	
,, 17	3.27 a.m.	6·07	10.21 a.m.	0·65	,, 17
	4.8 p.m.	5·46	10.18 p.m.	0·27	
,, 18	4.22 a.m.	5·38	*10.46 a.m.	0·24	,, 18
	*5.44 p.m.	5·00	10.54 p.m.	0·68	
,, 19	4.45 a.m.	5·56	11.0 a.m.	0·16	,, 19
	5.18 p.m.	4·85	11.45 p.m.	0·02	
,, 20	5.30 a.m.	4·27	11.43 a.m.	−0·21	,, 20
	6.14 p.m.	4·41			

Note.—The record for the ten days succeeding June 4 was quite unreliable owing to instrument being out of order.

At the points marked * the record also shows evidence of some instrumental friction or minor disturbance.

Heights are measured from P.E. Harbour Datum Mark.

Times are South African Civil Times (2 hours E. of Greenwich).

SCHEDULE IV.

Data Measured on Durban Tide Gauge Record, June, 1905.

High Water.				Low Water.			
Date.	S.A. Time.	Height.		S.A. Time.	Height.		Date.
		Ft.	In.		Ft.	In.	
June 2	3.34 a.m.	6	0	9.37 a.m.	1	3	June 2
	3.55 p.m.	5	10	9.36 p.m.	1	2	
,, 3	3.52 a.m.	6	2½	10.2 a.m.	0	9½	,, 3
	4.23 p.m.	6	0	10.8 p.m.	0	10½	
,, 4	4.20 a.m.	6	1	10.31 a.m.	0	8	,, 4
	4.48 p.m.	5	10	10.40 p.m.	0	9	
,, 5	4.58 a.m.	5	9	11.12 a.m.	0	5	,, 5
	5.23 p.m.	5	7½	11.9 p.m.	0	10	
,, 6	5.30 a.m.	5	9½	11.35 a.m.	0	10	,, 6
	5.47 p.m.	5	10½	11.57 p.m.	1	1	
,, 7	6.8 a.m.	5	5½	12.22 p.m.	0	10½	,, 7
	6.27 p.m.	5	5	12.25 a.m.	1	3	,, 8
,, 8	6.52 a.m.	5	1	12.56 p.m.	1	2	
	7.23 p.m.	5	3	1.36 a.m.	1	8½	,, 9
,, 9	7.35 a.m.	4	11	1.50 p.m.	1	8	
	8.15 p.m.	5	1½	2.27 a.m.	2	0½	,, 10
,, 10	8.41 a.m.	4	8½	2.53 p.m.	2	0	
	9.34 p.m.	4	9	3.32 a.m.	1	11	,, 11
,, 11	9.52 a.m.	4	2	4.27 p.m.	1	8	
	11.27 p.m.	4	7½	5.32 a.m.	1	5½	,, 12
,, 12	11.47 a.m.	4	1½	5.47 p.m.	1	4	
,, 13	12.11 a.m.	4	8½	6.37 a.m.	0	11½	,, 13
	1.0 p.m.	4	4	7.5 p.m.	0	10	
,, 14	1.16 a.m.	5	3	7.46 a.m.	0	10	,, 14
	2.0 p.m.	5	2	7.57 p.m.	1	0	
,, 15	2.16 a.m.	6	1	8.30 a.m.	1	0½	,, 15
	2.50 p.m.	5	10½	8.37 p.m.	0	11	
,, 16	3.2 a.m.	6	3½	9.3 a.m.	0	7½	,, 16
	3.38 p.m.	5	11½	9.32 p.m.	0	5	
,, 17	3.49 a.m.	5	11½	10.7 a.m.	0	1	,, 17
	4.12 p.m.	5	9½	9.47 p.m.	0	2½	
,, 18	4.25 a.m.	5	10	10.23 a.m.	0	1	,, 18
	4.43 p.m.	5	10	10.46 p.m.	0	3	
,, 19	4.58 a.m.	5	7	11.0 a.m.	0	1	,, 19
	5.30 p.m.	5	6	11.10 p.m.	0	3	
,, 20	5.38 a.m.	5	3½	11.40 a.m.	0	2	,, 20
	6.11 p.m.	5	5	12.5 a.m.	0	7½	,, 21
,, 21	6.20 a.m.	5	1	12.33 p.m.	0	6½	
	6.52 p.m.	5	2	12.30 a.m.	1	0	,, 22

All heights measured from Durban Harbour Datum Mark.

All times are expressed in South African Civil Time, or 2 hours fast on Greenwich M.T.

SCHEDULE V.

DATA MEASURED ON CAPE TOWN TIDE GAUGE RECORD, JUNE, 1905.

High Water.			Low Water.		
Date.	S.A. Time.	Height.	S.A. Time.	Height.	Date.
		Feet.		Feet	
June 4	4.55 a.m.	5·10	11.15 a.m.	1·03	June 4
	5.20 p.m.	4·95	11.30 p.m.	1·74	
,, 5	5.35 a.m.	5·20	11.40 a.m.	1·43	,, 5
	6.0 p.m.	5·00	12.20 a.m.	1·48	,, 6
,, 6	6.25 a.m.	4·78	12.45 p.m.	1·30	
	6.55 p.m.	4·65	1.15 a.m.	1·49	,, 7
,, 7	6.55 a.m.	4·42	12.55 p.m.	1·38	
	7.25 p.m.	4·53	2.20 a.m.	1·71	,, 8
,, 8	8.10 a.m.	4·29	2.30 p.m.	1·62	
	8.50 p.m.	4·61	3.10 a.m.	1·80	,, 9
,, 9	9.0 a.m.	4·15	3.30 p.m.	2·10	
	9.35 p.m.	4·50	4.15 a.m.	1·66	,, 10
,, 10	10.15 a.m.	4·00	4.40 p.m.	1·52	
	11.15 p.m.	4·46	5.35 a.m.	1·14	,, 11
,, 11	11.55 a.m.	3·93	5.50 p.m.	1·26	
,, 12	12.10 a.m.	4·32	6.35 a.m.	0·72	,, 12
	12.50 p.m.	4·10	6.45 p.m.	0·95	
,, 13	1.0 a.m.	4·56	7.50 a.m.	0·60	,, 13
	2.0 p.m.	4·39	7.40 p.m.	0·90	
,, 14	2.15 a.m.	4·92	8.15 a.m.	0·88	,, 14
	2.50 p.m.	5·00	9.5 p.m.	1·12	
,, 15	3.0 a.m.	5·36	9.30 a.m.	0·87	,, 15
	3.30 p.m.	5·26	10.0 p.m.	1·00	
,, 16	3.50 a.m.	5·18	9.50 a.m.	0·80	,, 16
	4.25 p.m.	5·19	10.25 p.m.	1·60	
,, 17	4.30 a.m.	5·40	10.45 a.m.	0·88	,, 17
	5.0 p.m.	5·09	11.15 p.m.	0·99	
,, 18	5.5 a.m.	4·64	11.25 a.m.	0·63	,, 18
	5.35 p.m.	4·58	11.20 p.m.	0·77	
,, 19	5.45 a.m.	4·24	11.40 a.m.	0·68	,, 19
	6.15 p.m.	4·25	12.5 a.m.	0·94	,, 20
,, 20	6.25 a.m.	4·00	12.20 p.m.	1·03	
	7.0 p.m.	4·38	1.45 a.m.	1·59	,, 21

Heights are measured from Table Bay Datum Mark.
Times are South African Civil Time (2 hours E. of Greenwich).

SCHEDULE VI.

Clock Rate Data for the Water-level Records of 1908, and Concurrent Barograms.

Week Record Sheet.		Lengths of Record in Inches.	Lengths of Record in Hours.	Deduced Rates in Inches per Hour.
Put on at—	Taken off at—			
6.40 a.m., March 2	7.20 a.m., March 9	12·1	168·6	·0717
7.20 a.m. ,, 9	8.0 a.m. ,, 16	12·2	168·6	·0724
8.0 a.m. ,, 16	9.5 a.m. ,, 23	12·1	169·1	·0715
9.6 a.m. ,, 23	6.50 a.m. ,, 30	12·0	165·7	·0724
6.50 a.m. ,, 30	8.0 a.m., April 6	12·15	169·2	·0718
8.5 a.m., April 6	8.30 a.m. ,, 13	12·1	168·4	·0718
10.30 a.m. ,, 13	10.0 a.m. ,, 20	12·1	167·5	·0722
10.0 a.m. ,, 20	9.40 a.m. ,, 27	12·05	167·6	·0719
9.45 a.m. ,, 27	10.35 a.m., May 4	12·15	168·8	·0719
10.35 a.m., May 4	8.20 a.m. ,, 12	12·16	189·7	·0717
8.20 a.m. ,, 12	9.15 a.m. ,, 18	10·5	144·9	·0724
9.15 a.m. ,, 18	9.30 a.m. ,, 25	12·15	168·3	·0722
9.35 a.m. ,, 25	9.25 a.m., June 1	11·95	167·8	·0712
9.25 a.m., June 1	1.10 p.m. ,, 8	12·4	171·7	·0722
1.10 p.m. ,, 8	9.50 a.m. ,, 15	11·95	164·7	·0725
Barograms.				
6.31 a.m., March 2	7.11 a.m., March 9	11·1	168·7	·0659
7.13 a.m. ,, 9	8.0 a.m. ,, 16	11·07	168·8	·0655
8.0 a.m. ,, 16	9.9 a.m. ,, 23	11·0	169·1	·0650
9.10 a.m. ,, 23	6.45 a.m. ,, 30	10·85	167·6	·0647
6.45 a.m. ,, 30	8.10 a.m., April 6	11·05	169·4	·0652
8.10 a.m., April 6	10.35 a.m. ,, 13	11·17	170·4	·0655
10.35 a.m. ,, 13	10.5 a.m. ,, 20	10·95	167·5	·0654
10.5 a.m. ,, 20	10.10 a.m. ,, 27	10·9	167·9	·0649
10.0 a.m. ,, 27	10.15 a.m., May 4	11·0	168·25	·0654
10.15 a.m., May 4	8.25 a.m. ,, 12	12·38	190·2	·0651
8.25 a.m. ,, 12	9.23 a.m. ,, 18	9·5	145·0	·0655
9.25 a.m. ,, 18	9.50 a.m. ,, 25	11·01	168·4	·0654
9.50 a.m. ,, 25	9.40 a.m., June 1	10·96	167·8	·0653
9.40 a.m., June 1	1.20 p.m. ,, 8	11·04	171·7	·0643
1.20 p.m. ,, 8	9.56 a.m. ,, 15	10·74	164·6	·0653

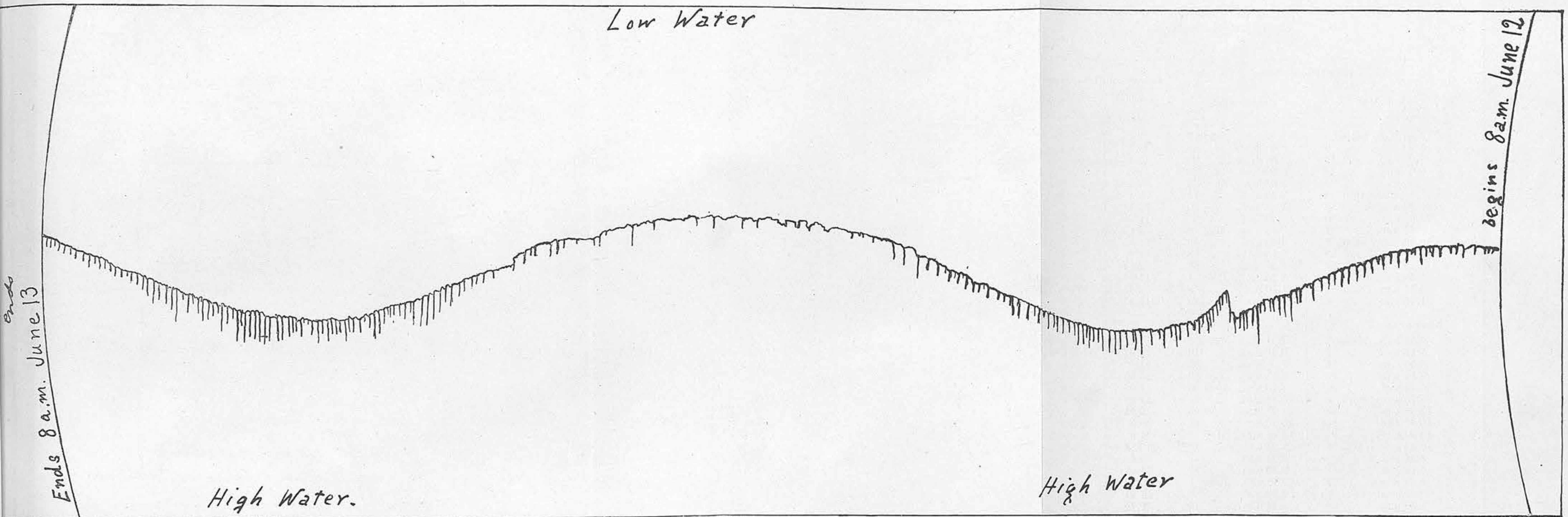

Low Water
begins 8 a.m. June 12
Ends 8 a.m. June 13
ends
High Water.
High Water

Fig. 4.

Automatic Records of Water Level in Bore-hole No. V and of Barometric Pressure at Tarka Bridge for the Period March 2 to June 15, 1908

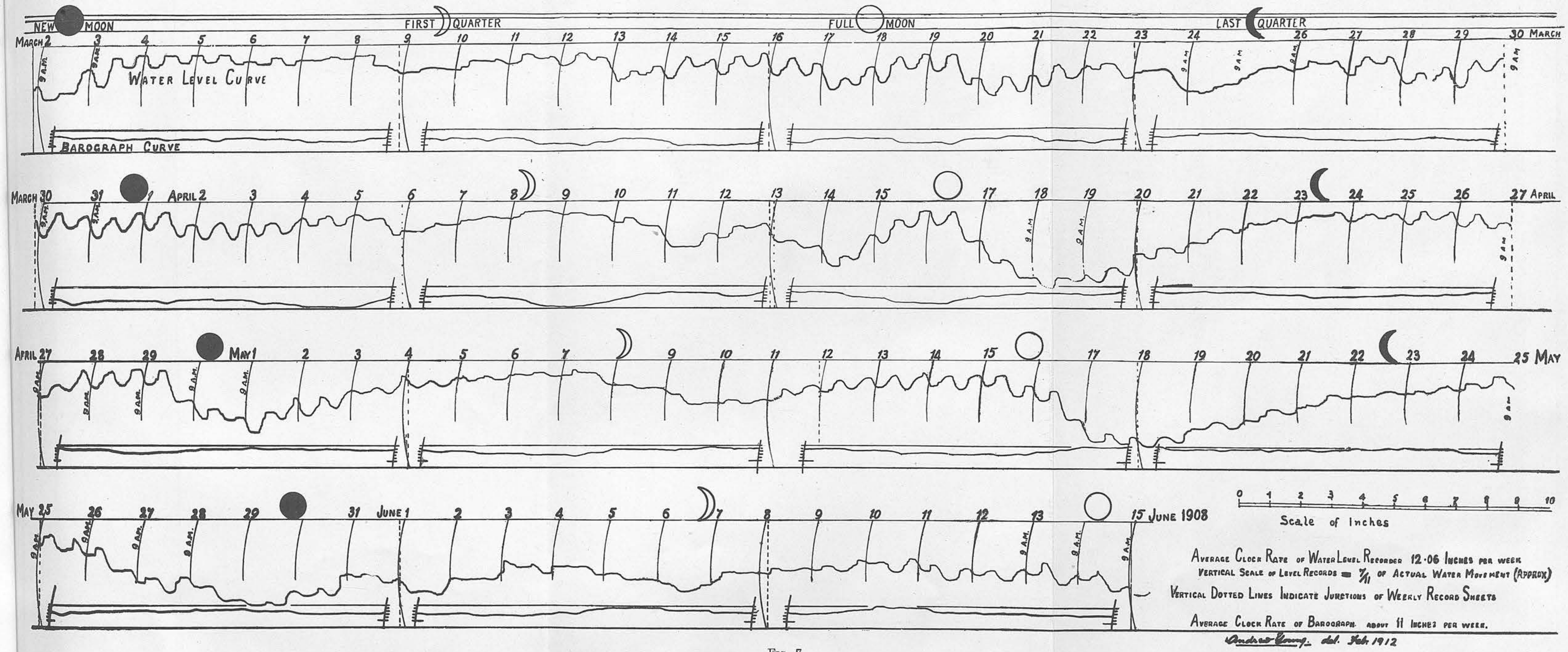
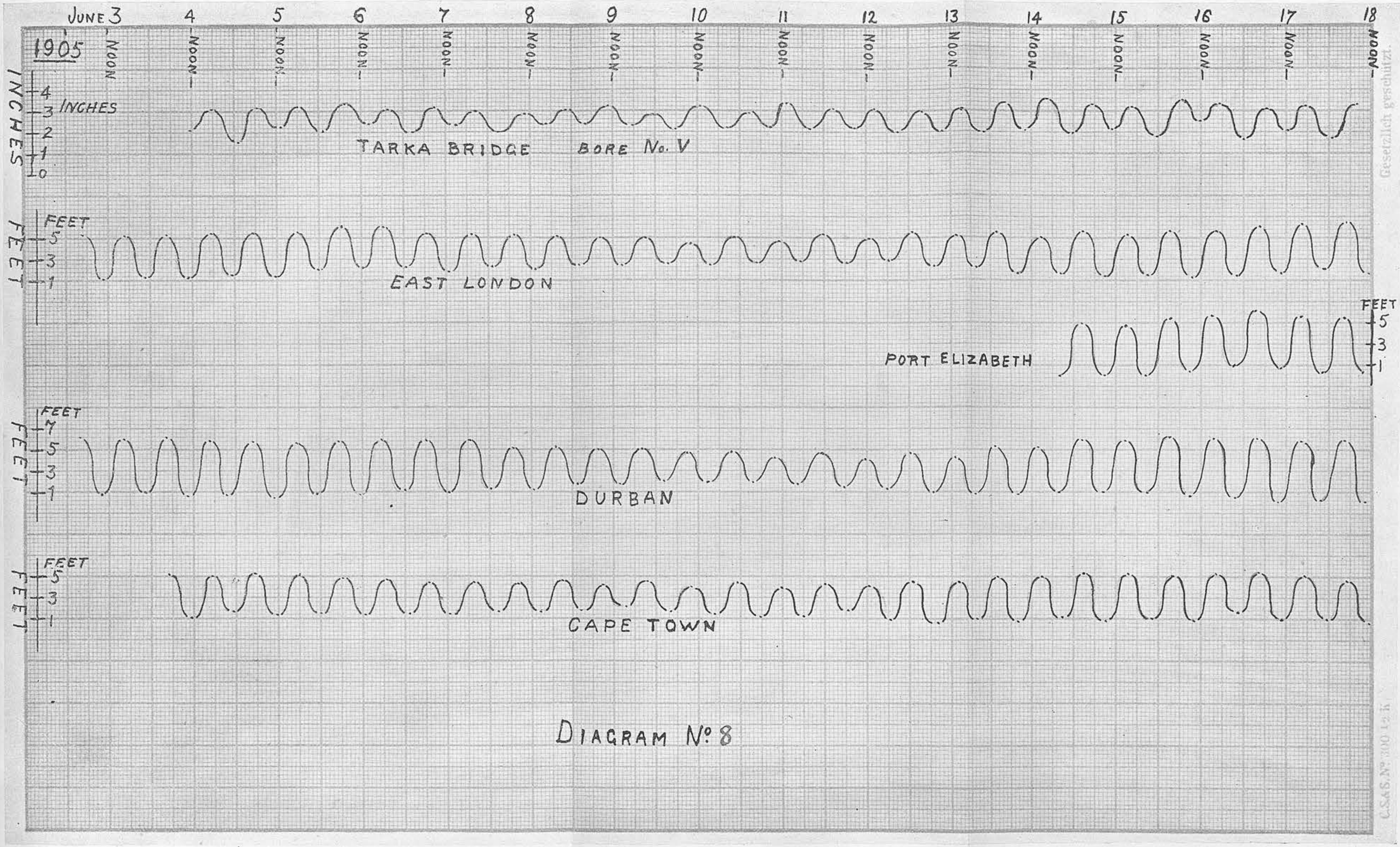


Fig. 7.



1905
JUNE 3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
NOON
INCHES
4
3
2
1
0
TARKA BRIDGE BORE No. V
FEET
5
3
1
EAST LONDON
PORT ELIZABETH
DURBAN
7
CAPE TOWN
DIAGRAM No. 8

Fig. 8.

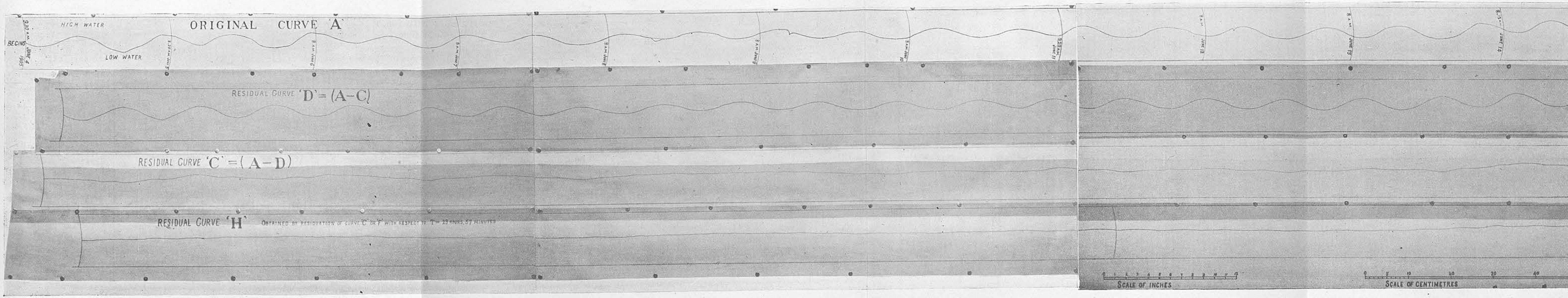

ORIGINAL CURVE 'A'
HIGH WATER
LOW WATER
BEGINS
RESIDUAL CURVE 'D' = (A-C)
RESIDUAL CURVE 'C' = (A-D)
RESIDUAL CURVE 'H'
SCALE OF INCHES
SCALE OF CENTIMETRES

Fig. 9.

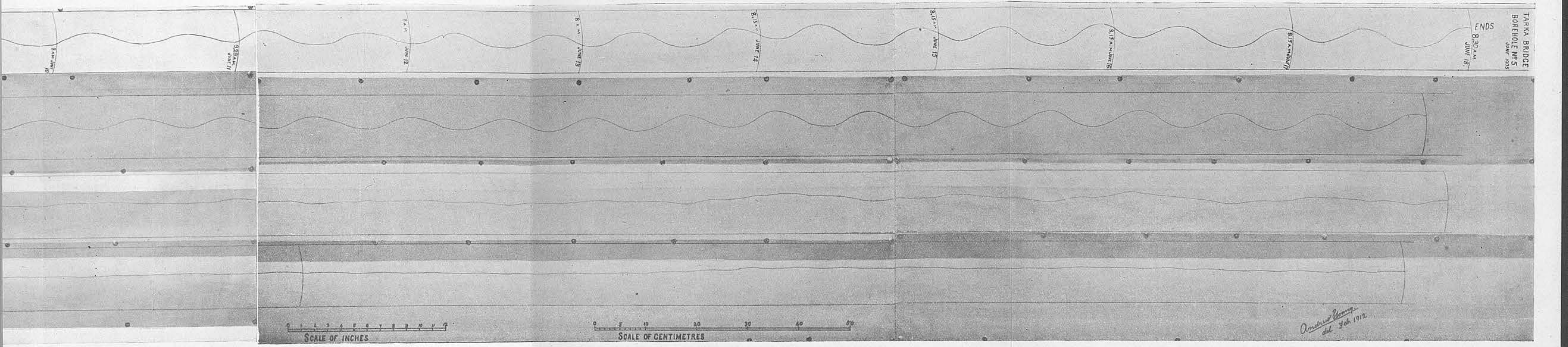

TARKA BRIDGE
BOREHOLE No. 5
JUNE 1905
ENDS
8.30 A.M.
JUNE 18.
SCALE OF INCHES
SCALE OF CENTIMETRES

Fig. 9.

APPENDIX.

As Chrystal's Residuation Method appears to be little known, and his original paper seems to be difficult of access in South Africa, I quote his description, which is as follows:—

"METHODS OF RESIDUATION.—In practice on Loch Earn, more particularly in our attempts to determine the positions of the nodes, we were compelled to work with short, large-scale limnograms; and the seiches were rarely pure. In these cases we resorted very often to a certain way of treating the limnogram, which we came ultimately to call 'Residuation.'

Consider a compound seiche, the equation of whose limnogram is—

$$y = A_1 \sin \frac{2\pi}{T_1}(t - a_1) + A_2 \sin \frac{2\pi}{T_2}(t - a_2) + \dots \text{ \&c.} \tag{18}$$

Construct a new curve by slipping the curve (18) a distance τ backwards along the t-axis, and from these two curves form a new one by adding the ordinates; or, what comes to the same thing, derive from (18) a new curve by adding to the ordinate at each point the ordinate of the point whose abscissa is greater by τ.

The equation of the resulting curve is—

$$\eta = 2A_1 \cos \frac{\pi\tau}{T_1} \sin \frac{2\pi}{T_1}\left(t - a_1 + \frac{\tau}{2}\right) + 2A_2 \cos \frac{\pi\tau}{T_2} \sin \frac{2\pi}{T_2}\left(t - a_2 + \frac{\tau}{2}\right) + .$$

The derived curve, or residual with respect to τ, contains in general all the harmonic oscillations of the original. The phase of each component is retarded by the same time, $\frac{1}{2}\tau$; and the amplitudes are altered in different proportions, viz.—

$$2 \cos (\pi\tau/T_1) : 1, \; 2 \cos (\pi\tau/T_2) : 1, \text{ \&c.}$$

Suppose, in particular, that we put $\tau = \frac{1}{2}T_1$; then the first component disappears altogether, and we get a curve, whose equation is—

$$\eta_1 = 2A_2 \cos \frac{\pi T_1}{2T_2} \sin \frac{2\pi}{T_2}\left(t - a_2 + \frac{T_1}{4}\right) + \text{, \&c.}$$

This last curve we call the residual of the limnogram with respect to the seiche of period T_1.

To show how this may be used in practice, suppose we have a short, large-scale limnogram, the principal or only components in which are the uninodal seiche (T_1) and the binodal (T_2). In general one, say the uninodal, will predominate; the other may be scarcely perceptible at first sight. Selecting two turning-points (points of symmetry, of course, if such are available), we divide the difference between the corresponding points by the number of intervening major oscillations. The result will be an approximation to T_1, say T'_1; but generally only an approximation, because the turning-points are displaced by the other seiches, mainly by the binodal.

Now residuate with respect to T'_1. The result will be a curve in which the uninodal harmonic is very much reduced, and will show the binodal in predominance. From this determine an approximation to T_2, say T'_2.

Now return to the original limnogram, and residuate with respect to T'_2. The result will be a curve from which the binodal harmonic is nearly absent. Hence the turning-points of the uninodal will be much less disturbed than before; and we can now get a better approximation to T_1, say T''_1.

Residuate now the original limnogram with respect to T''_1, and we get a curve in which the binodal is less disturbed than before. We can therefore make a closer approximation, say T''_2, to T_2; and so on.

The process can, if necessary, be repeated until the accumulated errors incidental to the manipulation obliterate the essential features of the diagram with which we are dealing.

In this way the periods of two or even three seiches can be determined, one after the other, from a comparatively short large-scale limnogram. We discovered in this way seiches the existence of which had not been suspected; and without this process we should not have obtained a determination of the period of the trinodal seiche of Earn at all, which never occurred pure, and had always a comparatively small range. The process was also used in purifying compound limnograms with a view to determine nodes" (Trans. Roy. Soc., Edinburgh, vol. xlv., part 2 (No. 14), p. 385).